GetMax

TRAINING ACADEMY

Formula and Cheat Sheets

Every formula, one table per topic, then the computed tables: factorials, nCr, dice, cards, sums, squares and cubes.

aptitude.getmaxsolutions.com · Every numeric answer in this book was checked by a program against the question before it was printed.

Start here: 3 easy methods that solve most questions

`Method 1: take 100`	When the question has no starting number, start with 100. Percent changes become plain rupees.
`Method 2: pick a convenient total`	When the job size or quantity is unknown, choose the LCM of the numbers given. Everything becomes a whole number.
`Method 3: try the options`	When setting up an equation feels hard, put each option into the question. The one that fits is the answer.
`Which method?`	Percent, profit, discount, price rise: Method 1. Time and work, pipes, ratio sharing: Method 2. Ages, numbers, 'find x': Method 3.

Where these shortcuts come from

`Urdhva-Tiryagbhyam - 'vertically and crosswise'`	Criss-cross multiplication of ANY two numbers. Each answer column = sum of the digit pairs whose places add up to that column.
`Nikhilam Navatashcaramam Dashatah - 'all from 9 and the last from 10'`	The base method for numbers near 10, 100, 1000. Also instant subtraction from a power of 10: 1000 - 357 → 9-3, 9-5, 10-7 = 643.
`Ekadhikena Purvena - 'by one more than the one before'`	Squares ending in 5: 85^2 → 8 × (8+1) | 25 = 7225.
`Antyayor Dashake'π - 'last digits adding to 10'`	63 × 67: same front, units add to 10 → 6 × 7 | 3 × 7 = 4221.
`Yavadunam - 'whatever the deficiency'`	Squares near a base: 96^2 → 96 - 4 = 92 | 4^2 = 16 → 9216.
`Anurupyena - 'proportionately'`	Working base: for numbers near 50 use 100/2, near 300 use 100 × 3.
`Ekanyunena Purvena - 'by one less than the one before'`	Times 9, 99, 999: 47 × 99 → 47 - 1 = 46 | 99 - 46 = 53 → 4653.
`Trachtenberg system (Jakow Trachtenberg, 1960)`	Digit-by-digit rules for x11, x12, x5, x9. The 'add the neighbour' rule for x11 is his.
`Lattice (gelosia) multiplication`	Medieval grid method, brought to Europe by Fibonacci. The same partial products as criss-cross, drawn in a grid.
`Casting out nines`	Centuries-old check: digit sums must agree. Catches most slips in 3 seconds.

Multiply without long multiplication

`2-digit × 2-digit (ab × cd)`	Right: b*d. Middle: a*d + b*c. Left: a*c. Write one digit, carry the rest.
`3-digit × 3-digit (abc × def)`	Five columns, right to left: cf | bf+ce | af+be+cd | ae+bd | ad.

`2-digit × 3-digit`	Put a zero in front: 45 becomes 045, then use the 3×3 pattern. The zero kills half the products.
`Near a base (100, 1000)`	Write each number as base +/- deviation. Left part = one number + other's deviation. Right part = product of deviations, padded to as many digits as the base has zeros.
`Same tens, units add to 10`	63 × 67: tens × (tens+1) | units × units → 6×7 | 3×7 = 4221.
$(a+b)(a-b) = a^2 - b^2$	$53 \times 47 = 50^2 - 3^2 = 2491$. Use it when two numbers sit equally either side of a round number.
`x 5, x 25, x 125`	x5 = x10 / 2. x25 = x100 / 4. x125 = x1000 / 8.
`x 9, x 99, x 999`	n × 99 = n × 100 - n.
`x 11`	Write the first digit, then each pair of neighbours added, then the last digit. Carry when a sum passes 9.

Squares, cubes and roots in seconds

`Ends in 5`	$(n5)^2 = n \times (n+1) \mid 25$. $85^2 = 8\times9 \mid 25 = 7225$.
`Near 50`	$(50 + k)^2 = (25 + k) \mid k^2$ (2 digits). $53^2 = 28\|09$. $47^2 = 22\|09$.
`Near 100`	$(100 + k)^2 = (100 + 2k) \mid k^2$. $96^2 = 92\|16$. $108^2 = 116\|64$.
Any 2-digit $(ab)^2$	$a^2 \mid 2ab \mid b^2$, then carry. $67^2 = 36 \mid 84 \mid 49 \rightarrow 4489$.
`Square-root last digit`	Square ends 1 → root ends 1 or 9. 4 → 2/8. 9 → 3/7. 6 → 4/6. 5 → 5. 0 → 0.
`Cube-root last digit`	Cube ends 2 → 8, 8 → 2, 3 → 7, 7 → 3; every other digit maps to itself.
`Learn by heart`	Squares 1-30, cubes 1-15, $2^1..2^{12}$, $3^1..3^7$. These show up inside every other topic.

Percent-fraction table and quick checks

`1/2 = 50%, 1/3 = 33.33%, 1/4 = 25%, 1/5 = 20%`	The base row.
`1/6 = 16.67%, 1/7 = 14.28%, 1/8 = 12.5%, 1/9 = 11.11%`	Learn the 7ths: 2/7 = 28.57, 3/7 = 42.85.
`1/11 = 9.09%, 1/12 = 8.33%, 1/15 = 6.67%, 1/16 = 6.25%`	1/16 is half of 1/8.
`3/8 = 37.5%, 5/8 = 62.5%, 7/8 = 87.5%`	Multiples of 12.5.
`Digit-sum (casting out 9s) check`	Digit sum of the product = digit sum of (digit-sum A × digit-sum B). If they differ, your answer is wrong.
`x% of y = y% of x`	16% of 25 = 25% of 16 = 4. Flip whenever it makes the number rounder.

Division, fractions and approximation under time pressure

`/ 5`	x 2, then / 10: 345 / 5 = 690 / 10 = 69.
`/ 25`	x 4, then / 100: 1150 / 25 = 4600 / 100 = 46.
`/ 125`	x 8, then / 1000: 3000 / 125 = 24000 / 1000 = 24.
`Compare a/b and c/d`	Cross-multiply: a × d vs b × c. The bigger product is on the side of the bigger fraction.
`1/7 = 0.142857 repeating`	2/7 = 0.285714, 3/7 = 0.428571: the same six digits, starting at a different place.
`BODMAS`	Brackets, Of, Division and Multiplication (left to right), Addition and Subtraction (left to right).
`Approximation`	Round to friendly numbers when options differ by more than 10%.

Number System

`Divisible by 3 / 9`	Digit sum divisible by 3 / 9.
`By 4 / 8`	Last 2 / last 3 digits divisible by 4 / 8.
`By 11`	(sum of odd-place digits) - (sum of even-place digits) is 0 or a multiple of 11.
`By 7`	Double the last digit and subtract it from the rest; repeat. 343 → 34 - 6 = 28, divisible.
`Unit digit cycles`	2: 2,4,8,6. 3: 3,9,7,1. 7: 7,9,3,1. 8: 8,4,2,6. 4 and 9 cycle in 2. 0,1,5,6 never change. Power mod 4, remainder 0 = last in the cycle.
`Remainder theorem`	Rem(a × b) = Rem(a) × Rem(b), reduced again. Find a power that leaves remainder 1 and use it to kill big exponents.
`HCF × LCM = a × b`	Two numbers only.
`HCF / LCM of fractions`	HCF = HCF(numerators)/LCM(denominators). LCM = LCM(numerators)/HCF(denominators).
`Trailing zeros of n!`	n/5 + n/25 + n/125 ... (whole parts only).
`Same remainder r for all divisors`	Number = k × LCM + r.

Percentage

`x% of y`	x/100 × y. Use the fraction table: 12.5% = 1/8, 16.67% = 1/6.
`A is r% more than B → B is less than A by`	r / (100 + r) × 100. (25% more → 20% less.)
`A is r% less than B → B is more than A by`	r / (100 - r) × 100. (20% less → 25% more.)
`Successive changes a% then b%`	a + b + ab/100 (use minus signs for decreases).

Price up r%, keep spend same	Cut consumption by r / (100 + r) × 100.
Population / depreciation	$P(1 + r/100)^n$, or $(1 - r/100)^n$ for decline.
Election, 2 candidates	Winning margin % of valid votes = winner% - loser%.

Profit and Loss

Profit % = (SP - CP)/CP × 100	Base is CP.
SP = CP × (100 + P%)/100	Loss: (100 - L%).
Same SP, x% gain on one, x% loss on other	Always a LOSS of $x^2/100$ %.
CP of a articles = SP of b articles	Profit % = (a - b)/b × 100.
False weight (sells at CP)	Gain % = error / (true - error) × 100. 900 g for 1 kg → 100/900.
Chain A→B→C	Final price = CP × (1 + p1)(1 + p2)...

Discount

SP = MP × (100 - d)/100	Discount is on MP.
Successive discounts a%, b%	Single equivalent = a + b - ab/100.
MP / CP = (100 + P%)/(100 - D%)	The one link between marked price and profit.
Buy x get y free	Discount % = y/(x + y) × 100.

Ratio and Proportion

Combine a:b and b:c	Make the common term equal: multiply across.
Mean proportional of a, b	√(ab).
Third proportional of a, b	b^2 / a.
Fourth proportional of a, b, c	bc / a.
Divide in 1/a : 1/b : 1/c	Multiply by the LCM of a, b, c to get whole numbers first.

Partnership

Share ratio	C1 × T1 : C2 × T2 : ...
Joins late / leaves early	Count only the months that money was actually in.
Withdraw or add mid-year	Split that partner into two blocks: (old capital × months) + (new capital × months).
Working partner	Take out the salary / commission first, split the rest by capital ratio.
Given profit ratio and time ratio	Capital ratio = profit / time for each.

Simple Interest

`SI = P × R × T / 100`	The one formula.
`Amount = P + SI`	
`Doubles in T years`	R = 100/T. Becomes n times in (n - 1) × 100 / R years.
`Two amounts at two times`	Yearly SI = difference / year gap. P = earlier amount - years × yearly SI.

Compound Interest

$A = P(1 + R/100)^n$	
`Half-yearly`	Rate R/2, periods 2n. Quarterly: R/4 and 4n. Monthly: R/12 and 12n.
`CI - SI, 2 years`	$P(R/100)^2$.
`CI - SI, 3 years`	$P R^2 (300 + R) / 100^3$.
`Doubles in n years`	Becomes 4x in 2n, 8x in 3n.
`Two amounts, consecutive years`	Rate = (later - earlier)/earlier × 100.

Time and Work

`Together`	1/a + 1/b = 1/T → T = ab/(a + b).
`LCM method`	Total work = LCM of days. Each person's rate = LCM / their days.
`Pairs given (A+B, B+C, C+A)`	Add all three rates and halve to get A+B+C.
`Men-days-hours`	M1 D1 H1 / W1 = M2 D2 H2 / W2.
`One leaves x days before the end`	Let total be T; the leaver worked T - x days.

Pipes and Cisterns

`Net rate`	Sum of 1/fill times - sum of 1/empty times.
`Leak`	1/leak = 1/normal - 1/with-leak.
`One closed after t minutes`	What the closed pipe did in t, plus what the rest do to finish, = 1.

Time and Distance

`km/h to m/s`	x 5/18. m/s to km/h: x 18/5.
`Average speed, equal distances`	2xy/(x + y). Never (x + y)/2.
`Fixed distance`	Speed ratio a:b → time ratio b:a.
`Late/early at two speeds`	d/s1 - d/s2 = total time gap.

`Speed x/y of usual, late by t`	Usual time = t × x/(y - x).
`Relative speed`	Same direction subtract, opposite directions add.

Problems on Trains

`Crossing a pole / man`	Time = train length / speed.
`Crossing platform / bridge`	Time = (train + platform) / speed.
`Two trains, opposite`	(L1 + L2)/(s1 + s2).
`Two trains, same direction`	(L1 + L2)/(s1 - s2).
`After meeting, take a and b hours to finish`	s1 : s2 = √(b) : √(a).

Boats and Streams

`Downstream`	b + s.
`Upstream`	b - s.
`Boat in still water`	(down + up)/2.
`Stream`	(down - up)/2.
`Round trip time`	d/(b + s) + d/(b - s).

Averages

`Average = sum / count`	
`Assumed mean (deviation) method`	Pick a round number, add the + and - gaps, divide by count, add back.
`Someone joins, average rises by d`	Newcomer = old average + (new count) × d.
`Someone leaves, average rises by d`	Leaver = old average - (new count) × d.
`Consecutive numbers`	Average = middle term = (first + last)/2.
`Overlapping groups (first 6, last 6 of 11)`	Middle term = sum of both groups - total.

Ages

`Age difference is constant`	If ratio changes from a:b to c:d, the difference in units must match: scale both ratios to the same difference.
`n years ago / hence`	Subtract / add n to EVERY person.
`Sum after n years`	Sum + n × (number of people).
`Children born at intervals`	Ages x, x + k, x + 2k ... sum = n × + k(0 + 1 + ... + n - 1).

Mixtures and Alligation

Alligation	Cheap qty : Dear qty = (Dear - Mean) : (Mean - Cheap).
Repeated removal and replacement	Pure left = Initial × $(1 - x/V)^n$.
Water added, sold at CP, gain g%	Water : Milk = g : 100.
Adding one component to shift ratio	Hold the other component fixed and rescale.

Mensuration

Rectangle	Area lb, perimeter 2(l + b), diagonal $\sqrt{l^2 + b^2}$.
Triangle (Heron)	s = (a + b + c)/2, area $\sqrt{s(s - a)(s - b)(s - c)}$.
Equilateral	$\sqrt{3}/4\ a^2$.
Circle	πr^2, circumference $2 \pi r$. Sector: (theta/360) πr^2.
Cube / cuboid	a^3, $6a^2$ / lbh, 2(lb + bh + hl). Cube diagonal $a\sqrt{3}$.
Cylinder	Volume $\pi r^2 h$, CSA $2 \pi r h$, TSA $2 \pi r(r + h)$.
Cone	Volume $(1/3) \pi r^2 h$, slant $l = \sqrt{r^2 + h^2}$, CSA $\pi r l$.
Sphere / hemisphere	$(4/3) \pi r^3$, $4 \pi r^2$ / $(2/3) \pi r^3$, $3 \pi r^2$.
% change of area	Each side changes x% → area changes $2x + x^2/100$ %.

Probability

P(E) = favourable / total	
P(not E) = 1 - P(E)	'At least one' is almost always 1 - P(none).
Addition	P(A or B) = P(A) + P(B) - P(A and B).
Independent	P(A and B) = P(A) × P(B).
Conditional	P(A | B) = P(A and B) / P(B).
Two dice	36 outcomes. Sum 7 has 6 ways, the most of any sum.
Cards	52 = 4 suits × 13. 12 face cards. 4 of each rank.

Permutation and Combination

Counting principle	Independent steps multiply (AND); alternatives add (OR).
nPr = n!/(n - r)!	Arrangements.
nCr = n!/(r!(n - r)!)	Selections.
Repeated letters	n!/(p! q! ...).
Circular	(n - 1)!. Necklace / garland (can flip): (n - 1)!/2.

`Items together`	Glue them into one block, arrange, then arrange inside the block.
`At least one of n items`	2^n - 1.
`Handshakes / lines`	nC2. Diagonals of an n-gon: nC2 - n.

Data Interpretation

`% growth`	(new - old)/old × 100.
`Share of total`	part/total × 100. For a pie: part/total × 360 degrees.
`Average across rows`	Total / count.
`Ratio`	Reduce by HCF; compare by cross-multiplying, not by dividing.

Series

`Differences`	Write the gaps. Constant → AP. Gaps in AP → second difference constant.
`Ratios`	x2, x3, or x2 +1 style (multiply then add).
`Squares / cubes`	n^2 +/- 1, n^3 +/- n. Memorise squares to 30 and cubes to 15.
`Alternate series`	Odd positions and even positions follow two different rules.
`Letter series`	Convert to positions (A = 1 ... Z = 26). EJOTY = 5, 10, 15, 20, 25.
`Wrong number`	Find the rule from the majority, then the odd term out.

Coding and Decoding

`Shift code`	Each letter moves +k (or -k). CAT → DBU is +1.
`Reverse alphabet`	A <→ Z: position becomes 27 - position.
`Number code`	Letters to positions, sometimes summed or multiplied.
`Word substitution`	'Sky is called sea' → answer with the NEW name.
`Chinese / sentence code`	Compare two sentences: the common word and the common code match.

Directions

`Right turn`	N → E → S → W → N (clockwise).
`Left turn`	N → W → S → E → N.
`Shortest distance`	√(net east^2 + net north^2).

`Shadow model`	Morning sun in the east → shadow to the WEST. Evening → shadow to the EAST.
`Angles`	Clockwise 90 = right, 180 = about-turn. 45 lands on NE/SE/SW/NW.

Analogy

`Word analogy`	Worker:tool, product:raw material, part:whole, animal:young, synonym, antonym.
`Number analogy`	Square, cube, x2+1, digit sum. 3:27 :: 4:64 (cube).
`Letter analogy`	Same shift on each letter. ACE:BDF :: GIK:HJL.

Ranking

`Total`	Left rank + right rank - 1.
`Swap two people`	The change in one's rank equals the change in the other's.
`Between two people`	Total - (sum of their ranks from opposite ends) ... draw it when overlap is possible.

Syllogisms

`All A are B`	A circle fully inside B.
`Some A are B`	Circles overlap (at least a bit).
`No A is B`	Circles apart.
`All + All = All`	All A are B, all B are C → all A are C.
`All + No = No`	All A are B, no B is C → no A is C.
`Some + All = Some`	Some A are B, all B are C → some A are C.
`Possibility`	'Some A being C is a possibility' follows whenever it is not ruled out by a 'No'.
`Either-or`	When two conclusions are 'Some X are Y' and 'No X is Y' and neither follows alone, 'either' follows.

Seating Arrangement

`Linear, facing north`	Your left = the reader's left.
`Circular, facing centre`	Seen from above: your left is clockwise, your right is anticlockwise.
`Immediate neighbours`	'Sits second to the right' means skip one seat.
`Count check`	n people round a table: each has exactly 2 neighbours; opposite exists only when n is even.

Statements: Assumptions and Conclusions

`Assumption test`	If the assumption were false, would the statement still make sense? If not, it is implicit.
`Conclusion test`	Must be true for ANY reader who accepts the statement. 'May', 'always', 'only' usually break it.
`Advertisement statements`	Assume people read ads and the product is worth buying.
`Extreme words`	'Only', 'all', 'never', 'best' in an option usually make it wrong.

Puzzles

`Grid`	Rows = people, columns = floors/days/colours.
`Definite first`	Place 'X lives on the top floor' clues before 'X is above Y' clues.
`Floor puzzles`	Floor 1 = bottom unless told otherwise. 'Above' does not mean 'immediately above'.
`Number puzzles`	Set up equations: sum, difference, product - then check options.

Odd One Out / Classification

`Numbers`	Prime / composite, square / cube, divisible by, digit sum.
`Letters`	Gaps between letters, vowels, reverse pairs.
`Words`	Category (fruit, tool), function, part-whole.

Blood Relations

`Father's / mother's brother`	Uncle (paternal / maternal).
`Brother's / sister's son`	Nephew; daughter → niece.
`Father's father`	Grandfather. Son's wife → daughter-in-law.
`'Only son of my father'`	The speaker himself.
`Coded relations`	A + B = A is father of B etc. Translate each symbol then draw.

Counting Figures

`Squares in an n × n grid`	$1^2 + 2^2 + ... + n^2 = n(n + 1)(2n + 1)/6$.
`Rectangles in an m × n grid`	$C(m + 1, 2) \times C(n + 1, 2)$.
`Triangle with n lines from one vertex to the base`	$n(n + 1)/2$ triangles (n lines including sides).
`Triangles in a triangle divided into n rows (all upright + inverted)`	For n = 1, 2, 3, 4: 1, 5, 13, 27.

Cryptarithmetic

`Leading carry`	When two n-digit numbers give an (n + 1)-digit sum, the new front digit is 1.
`Same letter, same digit`	Different letters, different digits. Leading letters are not 0.
`Column rule`	Digit + digit + carry-in = result + 10 × carry-out.
`X + X = X`	Then X = 0 (or X = 9 with carry-in 1 and a carry out).

Cheat sheets

Every number in these tables was computed by a program.

Factorials 0! to 15!

n! = n × (n-1) x ... x 2 × 1, and 0! = 1. Learn up to 8! by heart; the rest you can read off here. In exams you almost never multiply out a big factorial: cancel first. 10!/8! = 10 × 9 = 90.

n	n!	Trick to remember
0	1	Defined as 1 (one way to arrange nothing)
1	1	
2	2	
3	6	
4	24	24 hours in a day
5	120	120 = 5 × 4!
6	720	720 = degrees in 2 full turns
7	5,040	5040
8	40,320	40,320
9	3,62,880	3,62,880
10	36,28,800	36,28,800 (about 36 lakh)
11	3,99,16,800	
12	47,90,01,600	
13	6,22,70,20,800	
14	87,17,82,91,200	
15	13,07,67,43,68,000	

- Cancel before you multiply: 12!/(10! x 2!) = 12 × 11 / 2 = 66.
- n! = n × (n-1)!, so 9! = 9 × 8! = 9 × 40,320 = 3,62,880.
- Trailing zeros of n! = floor(n/5) + floor(n/25) + floor(n/125)... e.g. 100! has 20 + 4 = 24 zeros.

Combinations nCr: Pascal's triangle up to n = 12

nCr = n! / (r! (n-r)!) = the number of ways to CHOOSE r things from n when order does not matter. Each row of Pascal's triangle is nC0, nC1, ... nCn, and every number is the sum of the two above it.

Symmetry: nCr = nC(n-r). So 12C9 = 12C3 = 220; you never need r above n/2.

n	r=0	r=1	r=2	r=3	r=4	r=5	r=6
1	1	1					
2	1	2	1				

3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1
7	1	7	21	35	35	21	7
8	1	8	28	56	70	56	28
9	1	9	36	84	126	126	84
10	1	10	45	120	210	252	210
11	1	11	55	165	330	462	462
12	1	12	66	220	495	792	924

- nC0 = nCn = 1, nC1 = n, nC2 = n(n-1)/2.
- Quick nCr: top r numbers of n! divided by r!. 10C3 = (10 × 9 × 8)/(3 × 2 × 1) = 120.
- Sum of a whole row: nC0 + nC1 + ... + nCn = 2^n (number of subsets).

Permutations nPr (ordered choices)

nPr = n!/(n-r)! = n × (n-1) x ... (r numbers multiplied). Use it when ORDER matters: ranks, seats, passwords, president/secretary. nPr = nCr × r!.

n	r=1	r=2	r=3	r=4	r=5
1	1				
2	2	2			
3	3	6	6		
4	4	12	24	24	
5	5	20	60	120	120
6	6	30	120	360	720
7	7	42	210	840	2520
8	8	56	336	1680	6720
9	9	72	504	3024	15120
10	10	90	720	5040	30240

- Order matters? Use P. Order does not matter (team, committee, handshake, group)? Use C.
- 'Arrange all n' = nPn = n!.

Counting shortcuts (no listing by hand)

Each line is a pattern that turns a 'count them' question into one calculation.

Situation	Formula	Example

Arrange n different things in a row	n!	5 books: 5! = 120
Arrange with repeats (p alike, q alike...)	n! / (p! q! ...)	LETTER (T x2, E x2): 6!/(2! 2!) = 180
Around a circle (n people)	(n - 1)!	6 people: 5! = 120
Necklace / garland (can flip it)	(n - 1)! / 2	7 beads: 6!/2 = 360
Handshakes among n people	nC2 = n(n-1)/2	10 people: 45
Diagonals of an n-sided polygon	n(n - 3)/2	Hexagon: 9
Triangles from n points (no 3 in a line)	nC3	8 points: 56
Straight lines from n points (no 3 in a line)	nC2	8 points: 28
Subsets of a set with n items	2^n	5 items: 32
Non-empty selections	$2^n - 1$	4 fruits: 15
n identical things into r groups (any group may be empty)	(n + r - 1)C(r - 1)	10 sweets, 3 kids: 12C2 = 66
Same, every group gets at least one	(n - 1)C(r - 1)	10 sweets, 3 kids: 9C2 = 36
Letters in wrong envelopes (nobody right)	D(n): 0, 1, 2, 9, 44, 265	4 letters: 9
Rectangles on an m × n grid of squares	(m+1)C2 × (n+1)C2	Chessboard 8×8: 1296
Squares on an n × n grid	$1^2 + 2^2 + ... + n^2$	Chessboard: 204
Numbers of k digits from n digits, repeats allowed	n^k	4-digit PIN: 10^4 = 10000

- Two tasks one after another (AND) → multiply. Either one task or the other (OR) → add.
- 'At least one' → total minus 'none'. Always faster than adding the cases.

Dice: every probability you need

Two dice give 6 × 6 = 36 equally likely outcomes. The number of ways to get a sum s is s - 1 for s up to 7, and 13 - s after that. So 7 is the most likely sum (6 ways).

Sum of two dice	2	3	4	5	6	7	8	9	10	11	12
Ways (out of 36)	1	2	3	4	5	6	5	4	3	2	1
Probability	1/36	1/18	1/12	1/9	5/36	1/6	5/36	1/9	1/12	1/18	1/36

Event with two dice	Ways	Probability
Doublet (both the same)	6	1/6
At least one six	11	11/36
No six at all	25	25/36
Sum is even	18	1/2
Sum is a prime (2, 3, 5, 7, 11)	15	5/12

Sum is a multiple of 3	12	1/3
Sum greater than 9	6	1/6
Product is even	27	3/4
Difference is exactly 1	10	5/18

Symmetric around 10.5: sum s has the same ways as 21 - s.

Sum of three dice	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ways (out of 216)	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1

- One die: P(any given number) = 1/6, P(even) = 1/2, P(prime: 2, 3, 5) = 1/2.
- n dice: total outcomes 6^n. P(at least one six in n throws) = 1 - $(5/6)^n$.

Coins: exactly k heads

n coins give 2^n outcomes. The number with exactly k heads is nCk. So P(exactly k heads) = nCk / 2^n.

Divide the number in the cell by the outcomes. 4 coins, exactly 2 heads: 6/16 = 3/8.

Coins	Outcomes	0 heads	1 heads	2 heads	3 heads	4 heads	5 heads	6 heads
1	2	1	1					
2	4	1	2	1				
3	8	1	3	3	1			
4	16	1	4	6	4	1		
5	32	1	5	10	10	5	1	
6	64	1	6	15	20	15	6	1

- P(at least one head with n coins) = 1 - $1/2^n$.
- P(all same with n coins) = $2/2^n$.

A deck of 52 cards

4 suits × 13 ranks. Spades and clubs are black, hearts and diamonds are red. Face (court) cards are J, Q, K. Honour cards are A, J, Q, K (some papers include 10: say which you use).

Group	How many	P(one card drawn)
Any one suit (e.g. hearts)	13	1/4
Red cards	26	1/2
Black cards	26	1/2
Aces	4	1/13
Kings	4	1/13
Face cards (J, Q, K)	12	3/13

Red face cards	6	3/26
Face cards or aces	16	4/13
Number cards 2 to 10	36	9/13
King or heart (13 + 4 - 1)	16	4/13
Red king	2	1/26
Queen of spades	1	1/52

Total ways to draw 2 cards: 52C2 = 1326.

Two cards drawn together	Count	Probability
Both aces	4C2 = 6	1/221
Both red	26C2 = 325	25/102
Both the same suit	4 × 13C2 = 312	4/17
One red, one black	26 × 26 = 676	26/51
Both face cards	12C2 = 66	11/221

Probability rules on one page

Four rules solve almost every placement probability question.

Rule	Formula	Use it when
Basic	P(E) = favourable / total	All outcomes equally likely
Complement	P(not E) = 1 - P(E)	'At least one', 'not all', 'none'
OR (addition)	P(A or B) = P(A) + P(B) - P(A and B)	Either can happen; subtract the overlap
OR, mutually exclusive	P(A or B) = P(A) + P(B)	They cannot happen together
AND, independent	P(A and B) = P(A) × P(B)	One does not affect the other (coins, dice, with replacement)
AND, dependent	P(A and B) = P(A) × P(B given A)	Without replacement: the second draw has one fewer
Conditional	P(A given B) = P(A and B) / P(B)	'Given that...', 'if it is known...'
Odds in favour a : b	P = a / (a + b)	Odds against a : b means P = b / (a + b)
At least one of independent events	1 - (1-p1)(1-p2)...	'The problem is solved', 'at least one hits'
Exactly k successes in n tries	nCk p^k $(1-p)^{n-k}$	Repeated independent trials

Summations and series

Never add a long list by hand. These formulas give the total at once.

Sum	Formula	n = 10	n = 20	n = 50	n = 100
1 + 2 + ... + n	n(n+1)/2	55	210	1,275	5,050
$1^2 + 2^2 + ... + n^2$	n(n+1)(2n+1)/6	385	2,870	42,925	3,38,350
$1^3 + 2^3 + ... + n^3$	$[n(n+1)/2]^2$	3,025	44,100	16,25,625	2,55,02,500
First n odd numbers 1 + 3 + 5...	n^2	100	400	2,500	10,000
First n even numbers 2 + 4 + 6...	n(n+1)	110	420	2,550	10,100

Series	nth term	Sum of n terms
Arithmetic (AP): a, a+d, a+2d...	a + (n-1)d	n/2 × (first + last) = n/2 [2a + (n-1)d]
Geometric (GP): a, ar, ar^2...	$a\,r^{n-1}$	$a\,(r^n - 1)/(r - 1)$
Infinite GP with $\lvert r\rvert < 1$	-	a / (1 - r)
Average of an AP	-	(first + last)/2 = the middle term
Number of terms from a to l, step d	-	(l - a)/d + 1

- Sum of consecutive numbers = number of terms × average; the average is the middle term.
- Numbers from 1 to 100 divisible by 3: floor(100/3) = 33.

Squares, cubes and powers

Squares to 30 and cubes to 15 save time on every paper. Powers of 2 show up in coins, subsets and binary questions.

n	n^2	n	n^2	n	n^2
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	289	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

n	n^3	n	n^3	n	n^3
1	1	8	512	15	3,375
2	8	9	729	16	4,096

3	27	10	1,000	17	4,913
4	64	11	1,331	18	5,832
5	125	12	1,728	19	6,859
6	216	13	2,197	20	8,000
7	343	14	2,744	21	

k	2^k	3^k	5^k
1	2	3	5
2	4	9	25
3	8	27	125
4	16	81	625
5	32	243	3,125
6	64	729	15,625
7	128	2,187	78,125
8	256	6,561	3,90,625
9	512	19,683	
10	1,024	59,049	
11	2,048		
12	4,096		
13	8,192		
14	16,384		
15	32,768		
16	65,536		

- Square of a number ending in 5: $n5^2 = n(n+1)$ then 25. $65^2 = 42|25 = 4225$.
- A perfect square never ends in 2, 3, 7 or 8.
- Cube roots of perfect cubes: the last digit decides the last digit (2<→8, 3<→7 swap; the rest stay).

Fraction to percentage table

Know these and most percentage questions become one mental step.

Fraction	Percent	Fraction	Percent
1/1	100%	1/11	9.09%
1/2	50%	1/12	8.33%
1/3	33.33%	1/13	7.69%
1/4	25%	1/14	7.14%

1/5	20%	1/15	6.67%
1/6	16.67%	1/16	6.25%
1/7	14.29%	1/17	5.88%
1/8	12.5%	1/18	5.56%
1/9	11.11%	1/19	5.26%
1/10	10%	1/20	5%

- x% of y = y% of x. 16% of 25 = 25% of 16 = 4.
- A rise of 1/n then a fall back needs a fall of 1/(n+1).

Unit digits of powers (cyclicity)

Only the last digit of the base matters. Find the exponent's remainder when divided by 4 (use 4 when the remainder is 0).

Last digit	power 1	power 2	power 3	power 4	Repeats every
0	0	0	0	0	1
1	1	1	1	1	1
2	2	4	8	6	4
3	3	9	7	1	4
4	4	6	4	6	2
5	5	5	5	5	1
6	6	6	6	6	1
7	7	9	3	1	4
8	8	4	2	6	4
9	9	1	9	1	2

- 7^{95}: 95 leaves 3 when divided by 4, so the unit digit is the same as 7^3 → 3.
- 0, 1, 5, 6 always end in themselves.

Units, conversions and exam-hall tactics

Small conversions cause most silly mistakes. Keep these next to you.

Convert	How
km/h to m/s	x 5/18 (72 km/h = 20 m/s)
m/s to km/h	x 18/5 (15 m/s = 54 km/h)
1 hectare	10,000 m^2
1 litre	1000 cm^3

1 m^3	1000 litres
1 km	1000 m
1 hour	3600 seconds
π	22/7 (use when a radius is a multiple of 7), else 3.14

Under time pressure	**How it saves time**
Look at the options first	Often only one option has the right unit digit or size
Unit-digit check	Multiply only the last digits to rule options out
Digit-sum (casting out 9s) check	Digit sums of the question and answer must match
Approximate, then pick	Round 49.8% to 50%; pick the option nearest your estimate
Plug the options in	For ages and 'find the number', test the middle option first
Take 100 or the LCM	Turns percentages and work problems into whole numbers
Skip and return	Spend at most 90 seconds on a question in the first pass