GetMax
TRAINING ACADEMY

Aptitude Handbook

39 topics taught from zero: the idea in plain words, a diagram for each sub-topic, worked examples that say why each step happens, the shortcuts, the traps, and a 2-hour class plan.

aptitude.getmaxsolutions.com · Every numeric answer in this book was checked by a program against the question before it was printed.

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START HERE

Start here: 3 easy methods that solve most questions

> **You do not need to be 'good at maths' for placement aptitude. You need three habits: start from 100, pick a convenient total, and try the options.**

These three methods are the fastest way in for anyone who finds formulas scary. They work on percentages, profit and loss, discount, ratio, time and work, pipes, ages and number puzzles: more than half of a typical quantitative paper. Every later chapter points back to them.

Method 1: pretend the number is 100

Many questions never tell you the starting price, salary or population. They only talk in percentages. So choose the starting number yourself, and choose 100, because a percentage of 100 is just that many rupees.

Then apply each change in order, as plain numbers. At the end, compare with 100.

Start at 100. A 20% rise adds 20, not '20%'

Example 1: A shopkeeper raises a price by 25% and then gives a 20% discount. What is the overall change?

1. Start at 100.
   Why: No price is given, so we choose a friendly one.
2. 25% rise: 100 + 25 = 125.
   Why: 25% of 100 is 25.
3. 20% discount on 125: 20% of 125 = 25, so 125 - 25 = 100.
   Why: The discount is on the NEW price, not on 100.
4. Back to 100: no change.

Answer: 0% (no change)

Example 2: A's salary is 20% more than B's. By what percent is B's salary less than A's?

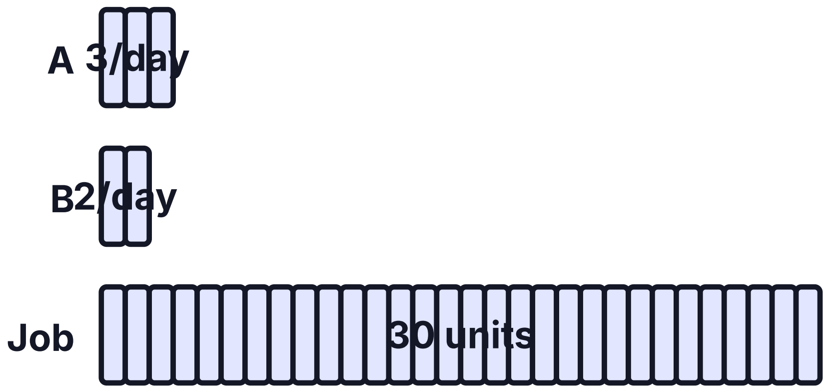
1. Let B = 100. Then A = 120.
   Why: The sentence compares A to B, so B is the base.
2. B is 20 less than A.
   Why: Difference stays 20.
3. As a percent of A: 20/120 × 100 = 16.67%.
   Why: 'Less than A' means A is now the base.

Answer: 16.67%

Method 2: pick a convenient total

When a job, a tank or an amount has no size, give it one. Choose the LCM (lowest common multiple) of the numbers in the question, so everyone's share per day comes out whole.

Now 'A finishes in 10 days' simply means 'A does 3 units a day' if the job is 30 units. Adding people is plain addition.



Job = LCM(10, 15) = 30 units. A does 3 a day, B does 2

Example 1: A can finish a job in 12 days and B in 18 days. How long will they take together?

1. Job = LCM(12, 18) = 36 units.
   Why: Both 12 and 18 divide 36, so no fractions.
2. A: 36/12 = 3 units a day. B: 36/18 = 2 units a day.
   Why: Units per day = total / days.
3. Together: 5 units a day.
   Why: Working together, their daily work adds.
4. 36 / 5 = 7.2 days.

Answer: 7.2 days

Method 3: try the options

Multiple-choice questions give you the answer, hidden among three wrong ones. When setting up an equation feels hard, put each option into the question and see which one fits.

Start with the middle option. If it is too big, the answer is one of the smaller options, so you only need one more try.

Example 1: The sum of two numbers is 45 and their difference is 9. The larger number is: (a) 25 (b) 27 (c) 29 (d) 31

1. Try 27: the other number is 45 - 27 = 18.
   Why: The sum fixes the other number.
2. Difference: 27 - 18 = 9. It fits.
   Why: Check the second condition.
3. Answer: 27.

Answer: 27

Exam tip: Trying options is not cheating. In an exam it is often the quickest correct method.

Formula summary

`Method 1: take 100`	When the question has no starting number, start with 100. Percent changes become plain rupees.
`Method 2: pick a convenient total`	When the job size or quantity is unknown, choose the LCM of the numbers given. Everything becomes a whole number.
`Method 3: try the options`	When setting up an equation feels hard, put each option into the question. The one that fits is the answer.
`Which method?`	Percent, profit, discount, price rise: Method 1. Time and work, pipes, ratio sharing: Method 2. Ages, numbers, 'find x': Method 3.

Shortcuts

Method 1: take 100 Use when: Percent, profit and loss, discount, successive increase or decrease.

1. Call the starting value 100.
2. Apply each change as plain rupees: +20% of 100 is +20.
3. Read the answer straight off. The final number minus 100 is the percent change.

Try it: A price goes up 20%, then down 20%. Net change?

1. Start at 100.
2. Up 20%: 100 + 20 = 120.
3. Down 20% of 120 = 24: 120 - 24 = 96.
4. 96 - 100 = -4, so a 4% fall.

Answer: 4% decrease

Method 2: pick a convenient total Use when: Time and work, pipes and cisterns, anything with 'together'.

1. Take the total work as the LCM of the days given.

2. Each person's work per day = total / their days.
3. Add the per-day work of people working together, then divide the total by it.

Try it: A finishes a job in 10 days, B in 15 days. Together?

1. LCM of 10 and 15 = 30 units of work.
2. A does 30/10 = 3 units a day. B does 30/15 = 2 units a day.
3. Together 5 units a day.
4. 30 / 5 = 6 days.

Answer: 6 days

Method 3: try the options Use when: Ages, 'find the number', ratio questions with awkward wording.

1. Start with the middle option so you learn whether to go higher or lower.
2. Check every condition in the question, not just one.
3. Stop at the first option that fits all of them.

Try it: A father is 3 times his son's age. In 10 years he will be twice the son's age. Son's age now? (a) 5 (b) 10 (c) 15 (d) 20

1. Try 10: father 30. In 10 years: 20 and 40. 40 = 2 × 20. Fits.
2. Both conditions are checked, so stop here.

Answer: 10 years

Common mistakes

Wrong: A 20% rise then a 20% fall brings you back to the start.	**Instead:** The fall is taken on the bigger number: 100 → 120 → 96, a 4% loss.
Wrong: In pipes questions, add every pipe's per-minute amount.	**Instead:** An emptying pipe SUBTRACTS its per-minute amount.
Wrong: The first option that fits the first sentence is the answer.	**Instead:** Check every condition in the question before you stop.

Check yourself

- You can start at 100 whenever a question gives only percentages.
- You can turn 'finishes in n days' into units per day using the LCM.
- You can try the options from the middle when the equation is awkward.

Class plan (2 hours)

`15 min`	Warm-up: five quick percentage questions done the long way; time them.
`25 min`	Method 1 with four examples: rise then fall, salary comparison, population change, price and spending.

`25 min`	Method 2: work, pipes, and sharing money, each with the LCM.
`20 min`	Method 3: options, with the 'start in the middle' rule.
`25 min`	Practice set on the site, timed.
`10 min`	Redo the warm-up with the methods; compare times. Recap.

Practice: 10 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/start-here-3-easy-methods-that-solve-most-questions

Where these shortcuts come from

> **Most of these tricks have names. The Indian set is called Vedic Mathematics: 16 short Sanskrit rules (sutras) published by Bharati Krishna Tirthaji in 1965. The German-Swiss set is the Trachtenberg system. Knowing the name tells students it is a real, general method, not a one-off party trick.**

Formula summary

`Urdhva-Tiryagbhyam - 'vertically and crosswise'`	Criss-cross multiplication of ANY two numbers. Each answer column = sum of the digit pairs whose places add up to that column.
`Nikhilam Navatashcaramam Dashatah - 'all from 9 and the last from 10'`	The base method for numbers near 10, 100, 1000. Also instant subtraction from a power of 10: 1000 - 357 → 9-3, 9-5, 10-7 = 643.
`Ekadhikena Purvena - 'by one more than the one before'`	Squares ending in 5: 85^2 → 8 × (8+1) | 25 = 7225.
`Antyayor Dashake'π - 'last digits adding to 10'`	63 × 67: same front, units add to 10 → 6 × 7 | 3 × 7 = 4221.
`Yavadunam - 'whatever the deficiency'`	Squares near a base: 96^2 → 96 - 4 = 92 | 4^2 = 16 → 9216.
`Anurupyena - 'proportionately'`	Working base: for numbers near 50 use 100/2, near 300 use 100 × 3.
`Ekanyunena Purvena - 'by one less than the one before'`	Times 9, 99, 999: 47 × 99 → 47 - 1 = 46 | 99 - 46 = 53 → 4653.
`Trachtenberg system (Jakow Trachtenberg, 1960)`	Digit-by-digit rules for x11, x12, x5, x9. The 'add the neighbour' rule for x11 is his.
`Lattice (gelosia) multiplication`	Medieval grid method, brought to Europe by Fibonacci. The same partial products as criss-cross, drawn in a grid.
`Casting out nines`	Centuries-old check: digit sums must agree. Catches most slips in 3 seconds.

Shortcuts

Nikhilam for subtraction Use when: Any 'power of ten minus a number'. Change counters, bills, 10000 - x.

1. Take every digit from 9, and the last digit from 10.
2. 10000 - 3786: 9-3, 9-7, 9-8, 10-6 = 6214.

Try it: 10000 - 3786

1. 9-3 = 6, 9-7 = 2, 9-8 = 1, 10-6 = 4.

Answer: 6214

Ekanyunena for × 99 Use when: Multiplying by 9, 99, 999 with the same number of digits.

1. Left: the number minus 1.
2. Right: the 9s minus the left part.

Try it: 47 × 99

1. Left 46.
2. Right 99 - 46 = 53.

Answer: 4653

Practice: 6 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/where-these-shortcuts-come-from

SPEED MATHS

Multiply without long multiplication

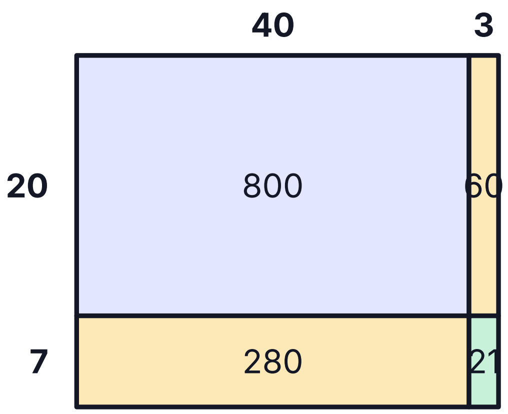
> **Long multiplication writes every partial product and adds them at the end. The shortcuts below do exactly the same sums, just in a smarter order, so you write only the answer.**

Every quantitative question ends in arithmetic. In a 60-minute paper with 30 questions you get 2 minutes each. Saving 20 seconds per multiplication is 5 extra minutes, which is 2 or 3 extra questions attempted. These are the speed tricks students asked for.

Why the tricks work: multiplication is area

43 × 27 is the area of a rectangle 43 wide and 27 tall. Split 43 into 40 + 3 and 27 into 20 + 7 and the rectangle falls into four pieces: 40 × 20, 3 × 20, 40 × 7 and 3 × 7.

Every multiplication trick is just a quick way to add those four pieces. Criss-cross adds the two middle pieces together in one step. Once you see the picture, you do not have to trust the trick, you can see why it must be right.



43 × 27: four pieces. Tens × tens, two cross pieces (the 'middle'), units × units. Total 1161

Example 1: 43 × 27 using the four pieces.

1. 40 × 20 = 800.
   Why: Tens × tens.
2. 3 × 20 = 60 and 40 × 7 = 280, together 340.
   Why: The two cross pieces: this is the 'middle' of criss-cross.
3. 3 × 7 = 21.
   Why: Units × units.
4. 800 + 340 + 21 = 1161.

Answer: 1161

Criss-cross: any two 2-digit numbers

Work right to left, writing one digit at a time and carrying the rest.

Right column: units × units. Middle column: the two cross products added (first digit of one × second digit of the other, both ways). Left column: tens × tens. Add each carry to the next column.

With practice this takes under 10 seconds. The animated visual further down the page shows it step by step.

`ab × cd`	right: b × d | middle: a × d + b × c | left: a × c (carry leftwards)

Example 1: 67 × 84

1. Right: 7 × 4 = 28. Write 8, carry 2.
   Why: Only one digit goes in each column.
2. Middle: 6 × 4 + 7 × 8 = 24 + 56 = 80, plus carry 2 = 82. Write 2, carry 8.
   Why: The two cross products are the middle pieces of the area picture.
3. Left: 6 × 8 = 48, plus carry 8 = 56. Write 56.
   Why: The last column takes the whole number.
4. Answer 5628.

Answer: 5628

Example 2: 38 × 76

1. Right: 8 × 6 = 48. Write 8, carry 4.
2. Middle: 3 × 6 + 8 × 7 = 18 + 56 = 74, + 4 = 78. Write 8, carry 7.
   Why: Always add the carry after the cross products.
3. Left: 3 × 7 = 21, + 7 = 28.
4. Answer 2888.

Answer: 2888

Exam tip: Say the carry out loud ('write 8, carry 4'). Dropped carries cause most mistakes, not wrong products.

Numbers close to 10, 100 or 1000 (base method)

When both numbers sit just below or just above a round number (the base), work with how far each is from the base (the deviation).

Left part: one number plus the OTHER number's deviation (both ways give the same answer). Right part: the two deviations multiplied, written with as many digits as the base has zeros.

Below the base the deviation is negative (97 is -3 from 100). Above, it is positive (104 is +4).

`97 × 96 (base 100)`	deviations -3 and -4. Left 97 - 4 = 93. Right (-3)(-4) = 12. Answer 9312.
`104 × 108`	+4 and +8. Left 104 + 8 = 112. Right 32. Answer 11232.
`103 × 98`	+3 and -2. Left 103 - 2 = 101. Right -6: borrow 1 hundred: 10100 - 6 = 10094.
`996 × 993 (base 1000)`	-4 and -7. Left 989. Right 028 (three digits). Answer 989028.

Example 1: 96 × 89

1. Base 100. Deviations: 96 is -4, 89 is -11.
   Why: Both are below 100.
2. Left: 96 - 11 = 85.
   Why: One number plus the other's deviation.
3. Right: (-4) × (-11) = 44, two digits.
   Why: Base 100 has two zeros, so the right part has two digits.
4. Answer 8544.

Answer: 8544

Example 2: 112 × 107

1. Base 100. Deviations +12 and +7.
2. Left: 112 + 7 = 119. Right: 12 × 7 = 84.
3. Answer 11984.

Answer: 11984

Exam tip: If the right part has more digits than the base has zeros, carry the extra digit into the left part: 88 × 88 → 76 | 144 → 7744.

Quick multipliers: x 5, x 25, x 125, x 9, x 99, x 11

x 5 is x 10 then halve. x 25 is x 100 then divide by 4. x 125 is x 1000 then divide by 8. Halving is easier than multiplying.

x 9 is x 10 minus the number. x 99 is x 100 minus the number: 47 × 99 = 4700 - 47 = 4653.

x 11 for a 2-digit number: write the first digit, then the sum of the two digits, then the last digit. 53 × 11 = 5 | 8 | 3 = 583. If the middle sum is 10 or more, carry: 78 × 11 = 7 | 15 | 8 → 858.

Multiply by	Do instead	Example
5	x 10, then / 2	86 × 5 = 860 / 2 = 430
25	x 100, then / 4	64 × 25 = 6400 / 4 = 1600
125	x 1000, then / 8	48 × 125 = 48000 / 8 = 6000
9	x 10, then minus the number	73 × 9 = 730 - 73 = 657
99	x 100, then minus the number	58 × 99 = 5800 - 58 = 5742
11	split and add neighbours	72 × 11 = 7 | 9 | 2 = 792
15	x 10, then add half of that	44 × 15 = 440 + 220 = 660
50	x 100, then / 2	37 × 50 = 3700 / 2 = 1850

Example 1: 264 × 25

1. 264 × 100 = 26400.
   Why: 25 is a quarter of 100.
2. 26400 / 4 = 6600.

Answer: 6600

Special pairs: same tens and units that add to 10, and (a + b)(a - b)

If two numbers have the same tens digit and their units add to 10 (63 and 67, 42 and 48), then: left part = tens × (tens + 1), right part = units × units (always two digits). 63 × 67 = 6 × 7 | 3 × 7 = 42 | 21 = 4221.

If two numbers sit the same distance either side of a round number, use $(a + b)(a - b) = a^2 - b^2$. $53 \times 47 = 50^2 - 3^2 = 2500 - 9 = 2491$.

Example 1: 84 × 86

1. Same tens 8, units 4 + 6 = 10.
   Why: The pattern fits.
2. Left 8 × 9 = 72. Right 4 × 6 = 24.
3. 7224.

Answer: 7224

Example 2: 98 × 102

1. Both are 2 away from 100.
   Why: Equal distance either side.
2. $100^2 - 2^2 = 10000 - 4 = 9996$.

Answer: 9996

Three-digit criss-cross

The same idea with five columns. For abc × def, the columns from the right are: c f | b f + c e | a f + b e + c d | a e + b d | a d.

The middle column has three products. For a 2-digit × 3-digit, put a 0 in front of the short number and use the same five columns: the zeros kill half the products.

Example 1: 234 × 567

1. c × f = 4 × 7 = 28. Write 8, carry 2.
2. b × f + c × e = 3 × 7 + 4 × 6 = 45, + 2 = 47. Write 7, carry 4.
3. a × f + b × e + c × d = 14 + 18 + 20 = 52, + 4 = 56. Write 6, carry 5.
   Why: The widest column: three pairs whose places add up to the same total.
4. a × e + b × d = 12 + 15 = 27, + 5 = 32. Write 2, carry 3.
5. a × d = 10, + 3 = 13.
6. Answer 1,32,678.

Answer: 132678

Exam tip: Check a 3-digit product with digit sums (see the checks chapter) before you mark the option.

Formula summary

`2-digit × 2-digit (ab × cd)`	Right: b*d. Middle: a*d + b*c. Left: a*c. Write one digit, carry the rest.
`3-digit × 3-digit (abc × def)`	Five columns, right to left: cf | bf+ce | af+be+cd | ae+bd | ad.
`2-digit × 3-digit`	Put a zero in front: 45 becomes 045, then use the 3×3 pattern. The zero kills half the products.
`Near a base (100, 1000)`	Write each number as base +/- deviation. Left part = one number + other's deviation. Right part = product of deviations, padded to as many digits as the base has zeros.
`Same tens, units add to 10`	63 × 67: tens × (tens+1) | units × units → 6×7 | 3×7 = 4221.
$(a+b)(a-b) = a^2 - b^2$	53 × 47 = $50^2 - 3^2$ = 2491. Use it when two numbers sit equally either side of a round number.
`x 5, x 25, x 125`	x5 = x10 / 2. x25 = x100 / 4. x125 = x1000 / 8.
`x 9, x 99, x 999`	n × 99 = n × 100 - n.
`x 11`	Write the first digit, then each pair of neighbours added, then the last digit. Carry when a sum passes 9.

Shortcuts

Criss-cross, 2 × 2 Use when: Any two 2-digit numbers. Target: under 10 seconds.

1. Units × units gives the last digit (carry the tens).
2. Cross: first-of-top × second-of-bottom + second-of-top × first-of-bottom, plus the carry.
3. Tens × tens plus the carry gives the front.

Try it: 43 × 27

1. Units: 3 × 7 = 21. Write 1, carry 2.
2. Cross: 4 × 7 + 3 × 2 = 34, plus 2 = 36. Write 6, carry 3.
3. Tens: 4 × 2 = 8, plus 3 = 11. Write 11.

Answer: 1161

Criss-cross, 3 × 3 Use when: Any two 3-digit numbers. Five columns instead of three.

1. Label abc × def.
2. Column 1: c*f.
3. Column 2: b*f + c*e.
4. Column 3: a*f + b*e + c*d (the big one, three products).
5. Column 4: a*e + b*d.
6. Column 5: a*d. Carry left at every step.

Try it: 234 × 567

1. c*f = 4×7 = 28 → write 8, carry 2.
2. b*f + c*e = 21 + 24 = 45, +2 = 47 → write 7, carry 4.
3. a*f + b*e + c*d = 14 + 18 + 20 = 52, +4 = 56 → write 6, carry 5.
4. a*e + b*d = 12 + 15 = 27, +5 = 32 → write 2, carry 3.
5. a*d = 10, +3 = 13 → write 13.

Answer: 1,32,678

2-digit × 3-digit Use when: Pad the short number with a leading zero and use the 3×3 columns.

1. 45 × 312 becomes 045 × 312.
2. Every product with the leading 0 vanishes, so it is quicker than a real 3×3.

Try it: 45 × 312

1. c*f = 5×2 = 10 → 0, carry 1.
2. b*f + c*e = 4×2 + 5×1 = 13, +1 = 14 → 4, carry 1.
3. a*f + b*e + c*d = 0 + 4 + 15 = 19, +1 = 20 → 0, carry 2.
4. a*e + b*d = 0 + 12 = 12, +2 = 14 → 4, carry 1.
5. a*d = 0, +1 = 1.

Answer: 14,040

Base 100 (both below) Use when: Both numbers between about 85 and 99.

1. Deviations from 100: 97 is -3, 96 is -4.
2. Left = 97 - 4 (or 96 - 3) = 93.
3. Right = (-3)(-4) = 12, written as 2 digits.
4. Answer 93|12.

Try it: 97 × 96

1. -3 and -4.
2. Left 93, right 12.

Answer: 9312

Base 100 (one above, one below) Use when: Numbers straddle 100.

1. 103 is +3, 98 is -2.
2. Left = 103 - 2 = 101 → 10100.
3. Right = (+3)(-2) = -6, so subtract.
4. 10100 - 6.

Try it: 103 × 98

1. 101|-06
2. = 10100 - 6

Answer: 10094

Base 1000 Use when: Numbers near 1000. Right part must be 3 digits.

1. 996 is -4, 993 is -7.
2. Left = 996 - 7 = 989.
3. Right = 28, padded to 028.

Try it: 996 × 993

1. 989|028

Answer: 9,89,028

Working base 50 Use when: Numbers near 50: use 100 as the reference, then halve.

1. 48 is -2, 47 is -3 from 50.
2. Cross: 48 - 3 = 45.
3. 45 × 50 = 2250 (half of 4500).
4. Add (-2)(-3) = 6.

Try it: 48 × 47

1. 45 × 50 = 2250
2. + 6

Answer: 2256

Times 11 Use when: Instant for any number.

1. Keep the outer digits, add neighbours in between.

Try it: 352 × 11

1. 3 | 3+5 | 5+2 | 2
2. 3 | 8 | 7 | 2

Answer: 3872

Common mistakes

Wrong: Forgetting the carry into the middle column.	**Instead:** Say the carry out loud: 'write 1, carry 2'. Most errors are dropped carries, not bad products.
Wrong: Writing 93|6 for a base-100 product with a 1-digit right part.	**Instead:** Pad the right part to as many digits as the base has zeros: 06, 006.
Wrong: Forgetting to add the carry in the middle column.	**Instead:** Say 'write, carry' at every column.
Wrong: Writing the base-method right part with the wrong number of digits (97 × 98 = 95 | 6 → 956).	**Instead:** Base 100 needs two digits: 95 | 06 = 9506.
Wrong: Using same-tens-units-add-to-10 on 63 × 68.	**Instead:** Check the pattern first: 3 + 8 is not 10. Use criss-cross instead.

Check yourself

- You can explain criss-cross with the four-piece area picture.
- You can multiply any two 2-digit numbers in under 15 seconds.
- You can use the base method for numbers near 100 or 1000.
- You can pick the fastest trick for × 5, 25, 125, 9, 99, 11.

Class plan (2 hours)

`10 min`	Race: two students multiply 43 × 27, one long way, one watching you do criss-cross.
`15 min`	The area picture. Draw it for 43 × 27 and point out the middle pieces.
`20 min`	Criss-cross 2 × 2: teach, then 8 drills in pairs.
`20 min`	Base method: both below, both above, mixed, base 1000.
`15 min`	Quick multipliers table and x 11. Students make their own examples.
`10 min`	Special pairs: same tens, and (a+b)(a-b).
`10 min`	3 × 3 criss-cross with the animated visual.
`15 min`	Speed drill sheet (PDF) against the clock.

`5 min`	Recap: which trick for which pair of numbers.

Practice: 25 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/multiply-without-long-multiplication

SPEED MATHS

Squares, cubes and roots in seconds

> **Squares and roots appear inside mensuration, number system, DI and simplification. Knowing four square patterns and one root trick saves a minute every time.**

Questions like 'square root of 7744' or 'value of 113^2' are asked directly, and squares hide inside Pythagoras, areas and compound interest. Learn squares to 30 by heart (cheat sheet) and use the patterns below for everything else.

Squares ending in 5

For a number n5 (like 35, 85, 115), multiply n by the next number, then write 25 after it.

Why: $(10n + 5)^2 = 100\,n^2 + 100\,n + 25 = 100\,n\,(n + 1) + 25$.

Example 1: 85^2

1. $8 \times 9 = 72$.
   Why: n times the next number.
2. Write 25 after: 7225.

Answer: 7225

Example 2: 115^2

1. $11 \times 12 = 132$.
   Why: n can have two digits.
2. 13225.

Answer: 13225

Squares near 50 and near 100

Near 50: for 50 + k, the first part is 25 + k and the last two digits are k^2. 53^2 = 28 | 09 = 2809. 47^2 = 22 | 09 = 2209.

Near 100: for 100 + k, the first part is 100 + 2k and the last two digits are k^2. 96^2 = 92 | 16 = 9216. 108^2 = 116 | 64 = 11664.

If k^2 has more than two digits, carry: 113^2 = 126 | 169 → 127 | 69 = 12769.

Example 1: 57^2

1. k = 7. First part 25 + 7 = 32.
   Why: Near 50 rule.
2. Last two digits $7^2 = 49$.
3. 3249.

Answer: 3249

Example 2: 93^2

1. k = -7. First part 100 - 14 = 86.
   Why: Near 100 rule; 2k = -14.
2. Last two digits 49.
3. 8649.

Answer: 8649

Any 2-digit square: a^2 | 2ab | b^2

For a two-digit number ab, write three blocks: a^2, then 2 × a × b, then b^2, and carry from right to left, one digit per block (except the left one).

67^2: 36 | 84 | 49 → write 9 carry 4; 84 + 4 = 88, write 8 carry 8; 36 + 8 = 44. Answer 4489.

Example 1: 73^2

1. Blocks: 49 | 42 | 9.
   Why: $a^2 = 49$, $2ab = 42$, $b^2 = 9$.
2. 9 stays. 42: write 2, carry 4. 49 + 4 = 53.
3. 5329.

Answer: 5329

Square roots of perfect squares

Step 1: the last digit of the square tells you two possible last digits of the root. 1 → 1 or 9, 4 → 2 or 8, 9 → 3 or 7, 6 → 4 or 6, 5 → 5, 0 → 0.

Step 2: cover the last two digits. The number left tells you the tens digit: find the biggest square below it.

Step 3: choose between the two candidates by comparing with the square of (tens digit)5.

Square ends in	1	4	5	6	9	0
Root ends in	1 or 9	2 or 8	5	4 or 6	3 or 7	0

Example 1: Square root of 7744

1. Ends in 4, so the root ends in 2 or 8.
   Why: $2^2 = 4$ and $8^2 = 64$ both end in 4.
2. Cover 44: 77 remains. $8^2 = 64 \le 77 < 81$, so the tens digit is 8.
   Why: The root is in the 80s.
3. $85^2 = 7225$, which is less than 7744, so take the larger candidate: 88.
   Why: Above the halfway square means the bigger ending.

Answer: 88

Cube roots of perfect cubes

Last digits of cubes are all different, so the last digit of a cube gives the last digit of the root directly. Only 2 <→ 8 and 3 <→ 7 swap; every other digit stays the same.

Cover the last three digits. The number left tells you the first digit: the biggest cube below it.

Cube ends in	0	1	2	3	4	5	6	7	8	9
Root ends in	0	1	8	7	4	5	6	3	2	9

Example 1: Cube root of 175616

1. Ends in 6, so the root ends in 6.
   Why: $6^3 = 216$ ends in 6.
2. Cover 616: 175 remains. $5^3 = 125 \le 175 < 216$, so the first digit is 5.
3. 56.

Answer: 56

Formula summary

`Ends in 5`	$(n5)^2 = n \times (n+1) \mid 25$. $85^2 = 8\times9 \mid 25 = 7225$.
`Near 50`	$(50 + k)^2 = (25 + k) \mid k^2$ (2 digits). $53^2 = 28 \mid 09$. $47^2 = 22 \mid 09$.
`Near 100`	$(100 + k)^2 = (100 + 2k) \mid k^2$. $96^2 = 92 \mid 16$. $108^2 = 116 \mid 64$.
`Any 2-digit (ab)`^{2}	$a^2 \mid 2ab \mid b^2$, then carry. $67^2 = 36 \mid 84 \mid 49 \to 4489$.
`Square-root last digit`	Square ends 1 → root ends 1 or 9. 4 → 2/8. 9 → 3/7. 6 → 4/6. 5 → 5. 0 → 0.
`Cube-root last digit`	Cube ends 2 → 8, 8 → 2, 3 → 7, 7 → 3; every other digit maps to itself.
`Learn by heart`	Squares 1-30, cubes 1-15, $2^1..2^{12}$, $3^1..3^7$. These show up inside every other topic.

Shortcuts

Square root of a 4-digit perfect square Use when: e.g. root of 7744.

1. Last digit 4 → root ends 2 or 8.
2. Front block 77 sits between 64 (8^2) and 81 (9^2) → tens digit 8.
3. Choose between 82 and 88: $85^2 = 7225 < 7744$, so the larger one.

Try it: Square root of 7744

1. Ends 2/8, tens 8, above 85^2.

Answer: 88

Cube root of a 6-digit perfect cube Use when: e.g. cube root of 175616.

1. Last digit 6 → root ends 6.
2. Strike the last three digits: 175. $5^3 = 125 \le 175 < 216 = 6^3$, so tens digit 5.

Try it: Cube root of 175616

1. 56.

Answer: 56

Check yourself

- You can square any number ending in 5, near 50 or near 100 in your head.
- You can find the square root of a 4-digit perfect square in 15 seconds.
- You can find the cube root of a 6-digit perfect cube in 10 seconds.

Class plan (2 hours)

`10 min`	Squares 11 to 30 round the room (cheat sheet).
`15 min`	Ending in 5, with the algebra on the board.
`20 min`	Near 50 and near 100, with carries.
`15 min`	$a^2 \mid 2ab \mid b^2$ for any two-digit square.
`20 min`	Square roots of 4-digit squares: three steps.
`15 min`	Cube roots of 6-digit cubes.
`20 min`	Practice set, timed.
`5 min`	Recap.

Practice: 17 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/squares-cubes-and-roots-in-seconds

SPEED MATHS

Percent-fraction table and quick checks

> **12.5% is 1/8. Once your brain reads it that way, '12.5% of 648' is just 648 / 8 = 81.**

Percentage, profit and loss, interest and data interpretation all turn into fractions. The table on the cheat-sheet page is worth learning cold. The checks at the end of this lesson catch wrong answers before you mark them.

Read percentages as fractions

Percent means 'out of 100'. 25% is 25/100 = 1/4. Most exam percentages are one of about twenty fractions, and dividing is faster than multiplying by a decimal.

Learn the family: halves, thirds, quarters, fifths, sixths, eighths, ninths, elevenths, twelfths. 1/8 = 12.5%, 3/8 = 37.5%, 5/8 = 62.5%, 7/8 = 87.5%.

Fraction	Percent	Fraction	Percent
1/2	50%	1/8	12.5%
1/3	33.33%	1/9	11.11%
1/4	25%	1/11	9.09%
1/5	20%	1/12	8.33%
1/6	16.67%	1/16	6.25%
1/7	14.28%	1/20	5%

Example 1: 37.5% of 1,288

1. 37.5% = 3/8.
   Why: From the table.
2. 1288 / 8 = 161.
   Why: Divide first.
3. 161 × 3 = 483.

Answer: 483

Example 2: 16.67% of 3,954

1. 16.67% = 1/6.
2. 3954 / 6 = 659.

Answer: 659

Two tricks that swap hard for easy

x% of y = y% of x. 18% of 50 is the same as 50% of 18 = 9. Flip whenever the flipped version is rounder.

Successive percentage changes a% then b%: the total change is a + b + ab/100 (use minus signs for falls). +20% then -10%: 20 - 10 - 200/100 = +8%.

Example 1: A price rises 10% and then another 10%. Total rise?

1. a + b + ab/100 = 10 + 10 + 100/100 = 21.
   Why: The second rise is on the bigger price, so there is an extra 1%.

Answer: 21%

Check your answer in three seconds

Digit-sum check (casting out nines): add the digits of each number until one digit is left (9 counts as 0). The digit sum of a product equals the digit sum of the two digit sums multiplied. If they differ, your answer is wrong.

Unit-digit check: the last digit of a product is the last digit of the two last digits multiplied. 487 × 36 must end in 7 × 6 = 42 → 2.

Size check: round both numbers and multiply. 487 × 36 is about 500 × 36 = 18,000, so 1,753 or 1,75,320 are impossible.

Example 1: Which is 487 × 36: (a) 17,532 (b) 17,542 (c) 17,432?

1. Unit digit: 7 × 6 = 42 → ends in 2. All three end in 2, so no help here.
   Why: Try the fastest check first.
2. Digit sums: 487 → 19 → 1. 36 → 9 → 0. 1 × 0 = 0.
   Why: Product's digit sum must be 0 (or 9).
3. 17532 → 18 → 0 passes. 17542 → 19 → 1 fails. 17432 → 17 → 8 fails.
4. (a).

Answer: 17,532

Formula summary

`1/2 = 50%, 1/3 = 33.33%, 1/4 = 25%, 1/5 = 20%`	The base row.
`1/6 = 16.67%, 1/7 = 14.28%, 1/8 = 12.5%, 1/9 = 11.11%`	Learn the 7ths: 2/7 = 28.57, 3/7 = 42.85.
`1/11 = 9.09%, 1/12 = 8.33%, 1/15 = 6.67%, 1/16 = 6.25%`	1/16 is half of 1/8.
`3/8 = 37.5%, 5/8 = 62.5%, 7/8 = 87.5%`	Multiples of 12.5.
`Digit-sum (casting out 9s) check`	Digit sum of the product = digit sum of (digit-sum A × digit-sum B). If they differ, your answer is wrong.
`x% of y = y% of x`	16% of 25 = 25% of 16 = 4. Flip whenever it makes the number rounder.

Shortcuts

Digit-sum check Use when: After any long multiplication, before you mark the option.

1. Reduce each factor to a single digit by adding digits (9 counts as 0).
2. Multiply those, reduce again.
3. Reduce your answer. They must match.

Try it: Is 234 × 567 = 132678?

1. 234 → 9 → 0. 567 → 18 → 0. 0 × 0 = 0.
2. 132678 → 27 → 0. Match, so it passes.

Answer: Passes

Percent as fraction in a question Use when: Whenever a % in the question is on the table.

1. Replace 37.5% by 3/8, then the number cancels cleanly.

Try it: 37.5% of 1,288

1. 1288 / 8 = 161, x 3 = 483.

Answer: 483

Check yourself

- You can read 12.5%, 16.67%, 37.5%, 83.33% as fractions without thinking.
- You can combine two successive % changes with a + b + ab/100.
- You can reject wrong options with digit-sum and unit-digit checks.

Class plan (2 hours)

`10 min`	Quiz: say each fraction as a percent, fast.
`25 min`	The table, with ten '% of' questions done by division.
`20 min`	x% of y = y% of x, and successive changes.
`25 min`	Checks: digit sum, unit digit, size.
`30 min`	Practice set, timed.
`10 min`	Recap.

Practice: 5 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/percent-fraction-table-and-quick-checks

Division, fractions and approximation under time pressure

> **Most students lose time dividing. Dividing by 25 is harder than multiplying by 4, so do the multiplication instead.**

Simplification and approximation questions ('approximately what value should replace ?') appear in most placement tests and bank-style papers. They reward three habits: turning division into multiplication, comparing fractions without long division, and rounding when the options allow it.

Divide by 5, 25 and 125 by multiplying

5 = 10/2, so dividing by 5 is the same as multiplying by 2 and dividing by 10. 25 = 100/4 and 125 = 1000/8 work the same way.

Dividing by 10, 100 or 1000 only moves the decimal point, so all the real work is a doubling or a × 4 or a × 8.

Example 1: 4,375 / 125

1. 4375 × 8 = 35,000.
   Why: 125 × 8 = 1000.
2. 35000 / 1000 = 35.

Answer: 35

Compare fractions without dividing

To compare a/b with c/d, cross-multiply: compare a × d with b × c. Whichever side has the bigger product has the bigger fraction.

For several fractions, convert to percentages with the fraction table, or compare how far each is from 1 (1 - a/b).

Example 1: Which is bigger: 13/17 or 16/21?

1. 13 × 21 = 273 and 17 × 16 = 272.
   Why: Cross products.
2. 273 > 272, so 13/17 is bigger (only just).

Answer: 13/17

BODMAS without mistakes

Order: Brackets, then 'of', then Division and Multiplication from left to right, then Addition and Subtraction from left to right.

Division does NOT come before multiplication. They share one level and you go left to right. 12 / 3 × 2 = 4 × 2 = 8, not 12 / 6 = 2.

Example 1: Simplify 48 / 4 × 3 + 5 - 2 × 6

1. 48 / 4 = 12, then 12 × 3 = 36.
   Why: Division and multiplication left to right.
2. 2 × 6 = 12.
3. 36 + 5 - 12 = 29.

Answer: 29

Approximation: when the options let you round

If the answer options are more than about 10% apart, you do not need the exact value. Round every number to the nearest friendly value, compute, and pick the nearest option.

Round in opposite directions when you can (one up, one down) so the errors cancel.

Example 1: Approximately: 399.8 × 24.9 / 50.1 = ? (a) 150 (b) 200 (c) 250 (d) 300

1. 400 × 25 / 50.
   Why: Friendly numbers.
2. = 200.

Answer: 200

Exam-hall time plan

Give each question at most 90 seconds on the first pass. If you are not near the answer by then, mark it and move on.

Look at the options before calculating: the unit digit, the size or the form of the answer often rules out two of them.

Do the easy topics of your paper first (percentages, averages, series), then return to the long ones (DI, puzzles).

Situation	What to do
Options far apart	Approximate
Options close together	Compute exactly, check the unit digit
'Find the number' type	Try the options from the middle
No starting value given	Take 100 or the LCM
Stuck after 90 s	Mark, skip, come back

Formula summary

/ 5	x 2, then / 10: 345 / 5 = 690 / 10 = 69.
/ 25	x 4, then / 100: 1150 / 25 = 4600 / 100 = 46.
/ 125	x 8, then / 1000: 3000 / 125 = 24000 / 1000 = 24.

`Compare a/b and c/d`	Cross-multiply: a × d vs b × c. The bigger product is on the side of the bigger fraction.
`1/7 = 0.142857 repeating`	2/7 = 0.285714, 3/7 = 0.428571: the same six digits, starting at a different place.
`BODMAS`	Brackets, Of, Division and Multiplication (left to right), Addition and Subtraction (left to right).
`Approximation`	Round to friendly numbers when options differ by more than 10%.

Common mistakes

Wrong: Doing all divisions before multiplications (12 / 3 × 2 = 2).	**Instead:** Same level, left to right: 12 / 3 = 4, then x 2 = 8.
Wrong: Computing exactly when the options are 50 apart.	**Instead:** Round and move on; save the time for harder questions.

Check yourself

- You can divide by 5, 25 and 125 without long division.
- You can compare two fractions by cross-multiplying.
- You can apply BODMAS left to right for / and x.
- You can decide from the options whether to approximate.

Class plan (2 hours)

`10 min`	Warm-up: 5 divisions by 25 and 125, long way vs the trick.
`20 min`	Division by multiplying.
`20 min`	Comparing fractions: cross-multiplying and the % table.
`15 min`	BODMAS traps.
`20 min`	Approximation: 10 questions with options.
`10 min`	The exam-hall time plan.
`20 min`	Practice set, timed.
`5 min`	Recap.

Practice: 9 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/division-fractions-and-approximation-under-time-pressure

Number System

> **Most number questions never need the whole number. 'Is it divisible by 8?' needs the last 3 digits. 'What is the unit digit?' needs the last digit. Learn which part to look at.**

Number system is the most common quant topic in placement papers: divisibility, unit digit, remainders, factors, HCF and LCM. Each one has a rule that avoids big calculations, and those rules are also used inside every other chapter.

Kinds of numbers

Natural numbers: 1, 2, 3 ... Whole numbers: 0, 1, 2, 3 ... Integers add the negatives: ... -2, -1, 0, 1, 2 ...

A prime has exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 ... 1 is NOT prime, and 2 is the only even prime. A composite number has more than two factors (4, 6, 8, 9 ...).

To test if a number up to 400 is prime, divide only by primes up to its square root. For 221: √(221) is about 14.9, so try 2, 3, 5, 7, 11, 13. 13 × 17 = 221, so it is not prime.

Co-primes are two numbers whose only common factor is 1, like 8 and 15. They do not have to be prime themselves.

Primes below 100 (25 of them)
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Example 1: Is 391 a prime number?

1. √(391) is just under 20, so test primes up to 19.
   Why: If 391 had a factor bigger than 20, it would also have one smaller than 20.
2. 2, 3, 5, 7, 11, 13: none divides. 17 × 23 = 391.
   Why: Check each prime in turn.
3. Not prime: 391 = 17 × 23.

Answer: Not prime

Divisibility rules

You can tell whether a number divides another without dividing. Each rule looks at only part of the number.

2, 5, 10: look at the last digit. 4: last two digits. 8: last three digits. 3 and 9: add all the digits. 11: alternate plus and minus across the digits; the result must be 0 or a multiple of 11.

For a composite divisor, split it into co-prime parts: divisible by 12 means divisible by 3 AND 4; by 72 means by 8 AND 9.

Divisor	Test	Example
2	last digit even	3,758
3	digit sum divisible by 3	4,212 (sum 9)

4	last two digits divisible by 4	7,316 (16)
5	last digit 0 or 5	8,275
6	divisible by 2 and 3	5,334
8	last three digits divisible by 8	91,128 (128)
9	digit sum divisible by 9	6,705 (sum 18)
11	alternating sum 0 or multiple of 11	8,261 (8-2+6-1 = 11)
12	divisible by 3 and 4	4,332
7	double the last digit, subtract from the rest; repeat	343 → 34 - 6 = 28

Example 1: Find the smallest digit * so that 5*2 is divisible by 6.

1. Divisible by 6 means divisible by 2 and by 3.
   Why: 6 = 2 × 3, and 2 and 3 are co-prime.
2. It ends in 2, so it is already even.
3. Digit sum 5 + * + 2 = 7 + * must be divisible by 3, so * = 2, 5 or 8.
4. Smallest is 2.

Answer: 2

Example 2: Is 7,29,432 divisible by 72?

1. 72 = 8 × 9, co-prime parts.
   Why: Check both rules; both must pass.
2. Last three digits 432 / 8 = 54, yes.
   Why: Rule for 8.
3. Digit sum 7+2+9+4+3+2 = 27, divisible by 9, yes.
   Why: Rule for 9.
4. Yes.

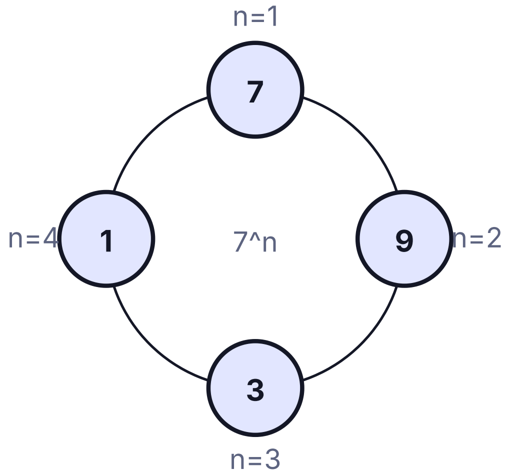
Answer: Yes

Unit digit of big powers

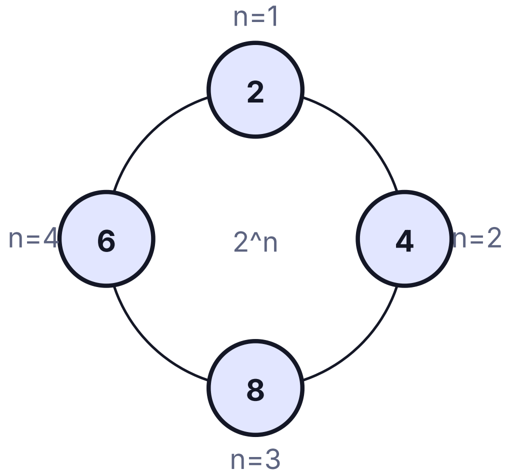
The last digit of a product depends only on the last digits of the numbers. And the last digits of powers repeat in a cycle.

2, 3, 7, 8 repeat every 4 powers. 4 and 9 repeat every 2. 0, 1, 5, 6 never change.

Method: divide the power by 4 and look at the remainder r. The unit digit is the same as for the power r (if r = 0, use power 4).



Unit digits of 7^n repeat every 4: 7, 9, 3, 1



Unit digits of 2^n repeat every 4: 2, 4, 8, 6

Example 1: Unit digit of 7^{95}.

1. 7's cycle: 7, 9, 3, 1 (length 4).
   Why: $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, then it repeats.
2. $95 = 4 \times 23 + 3$, remainder 3.
   Why: Full cycles do not change the position.
3. Third in the cycle: 3.

Answer: 3

Example 2: Unit digit of $2^{53} \times 3^{62}$.

1. $53 = 4 \times 13 + 1$, so 2^{53} ends like $2^1 = 2$.
2. $62 = 4 \times 15 + 2$, so 3^{62} ends like $3^2 = 9$.
3. $2 \times 9 = 18$, unit digit 8.

Answer: 8

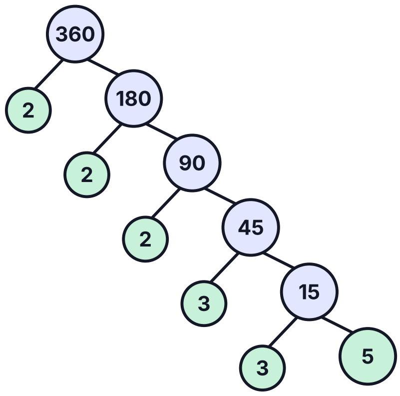
Exam tip: Write the four-cycles of 2, 3, 7 and 8 on your rough sheet once. Every unit-digit question is then 10 seconds.

Factors, and how many a number has

Factorise into primes first, using a factor tree. $360 = 2^3 \times 3^2 \times 5$.

Number of factors: add 1 to each power and multiply. For 360 that is $(3+1)(2+1)(1+1) = 24$.

Why: a factor of 360 picks 2 zero, one, two or three times (4 choices), 3 zero, one or two times (3 choices), and 5 zero or one time (2 choices).



360 = 2 × 2 × 2 × 3 × 3 × 5

Example 1: How many factors does 720 have?

1. $720 = 2^4 \times 3^2 \times 5$.
   Why: Prime factorisation.
2. $(4+1)(2+1)(1+1) = 5 \times 3 \times 2 = 30$.
   Why: Add one to each power, multiply.

Answer: 30

HCF and LCM

HCF (highest common factor) is the biggest number that divides all of them. LCM (lowest common multiple) is the smallest number all of them divide.

From prime factors: HCF takes the LOWEST power of each common prime; LCM takes the HIGHEST power of every prime present.

For two numbers only: HCF × LCM = product of the numbers.

Word clues: 'largest', 'greatest', 'maximum size of tiles' → HCF. 'Smallest', 'least', 'when will they meet again', 'bells ring together' → LCM.

`HCF × LCM = a × b`	Two numbers only.
`Largest number dividing a, b, c leaving the same remainder`	HCF of the differences (b - a), (c - b).
`Smallest number leaving remainder r when divided by x, y, z`	LCM(x, y, z) + r.
`HCF of fractions = HCF of numerators / LCM of denominators`	LCM of fractions = LCM of numerators / HCF of denominators.

Example 1: Three bells ring every 12, 15 and 20 minutes. They ring together at 9:00. When do they next ring together?

1. They ring together at common multiples of 12, 15 and 20.
   Why: 'Together again' is an LCM clue.
2. LCM(12, 15, 20) = 60 minutes.
   Why: $12 = 2^2 \times 3$, $15 = 3 \times 5$, $20 = 2^2 \times 5 \rightarrow 2^2 \times 3 \times 5$.
3. At 10:00.

Answer: 60 minutes

Example 2: Find the greatest number that divides 43, 91 and 183 leaving the same remainder.

1. Differences: 91 - 43 = 48, 183 - 91 = 92.
   Why: If both leave remainder r, the remainder cancels in the difference.
2. HCF(48, 92) = 4.
3. 4.

Answer: 4

Remainders

The remainder of a sum or product can be found from the remainders of the parts. 17 × 23 / 12: 17 leaves 5 and 23 leaves 11, 5 × 11 = 55, which leaves 7.

For big powers, find a small power that leaves remainder 1, then use it. $2^3 = 8$ leaves 1 when divided by 7, so $2^{99} = (2^3)^{33}$ leaves 1 too.

Negative remainders are allowed and often faster: 23 leaves -1 when divided by 12.

Example 1: Remainder when 2^{100} is divided by 7.

1. $2^3 = 8 = 7 + 1$ leaves remainder 1.
   Why: Look for a power just above a multiple of 7.
2. $2^{100} = (2^3)^{33} \times 2$.
   Why: $100 = 3 \times 33 + 1$.
3. Remainder = $1^{33} \times 2 = 2$.
   Why: Each 2^3 contributes a remainder of 1.

Answer: 2

Example 2: Remainder when 25^{25} is divided by 26.

1. 25 = 26 - 1 leaves remainder -1.
   Why: A negative remainder is often easier.
2. $(-1)^{25} = -1$.
   Why: Odd power keeps the minus sign.
3. -1 means 26 - 1 = 25.

Answer: 25

Trailing zeros of n!

A zero at the end comes from a factor 10 = 2 × 5. In n! there are always more 2s than 5s, so count the 5s.

Zeros in n! = n/5 + n/25 + n/125 + ... (drop the remainders each time).

Example 1: How many zeros does 100! end with?

1. 100/5 = 20.
   Why: Every multiple of 5 gives one 5.
2. 100/25 = 4.
   Why: Multiples of 25 give a second 5.
3. 20 + 4 = 24.

Answer: 24

Formula summary

`Divisible by 3 / 9`	Digit sum divisible by 3 / 9.
`By 4 / 8`	Last 2 / last 3 digits divisible by 4 / 8.
`By 11`	(sum of odd-place digits) - (sum of even-place digits) is 0 or a multiple of 11.
`By 7`	Double the last digit and subtract it from the rest; repeat. 343 → 34 - 6 = 28, divisible.
`Unit digit cycles`	2: 2,4,8,6. 3: 3,9,7,1. 7: 7,9,3,1. 8: 8,4,2,6. 4 and 9 cycle in 2. 0,1,5,6 never change. Power mod 4, remainder 0 = last in the cycle.
`Remainder theorem`	Rem(a × b) = Rem(a) × Rem(b), reduced again. Find a power that leaves remainder 1 and use it to kill big exponents.
`HCF × LCM = a × b`	Two numbers only.
`HCF / LCM of fractions`	HCF = HCF(numerators)/LCM(denominators). LCM = LCM(numerators)/HCF(denominators).
`Trailing zeros of n!`	n/5 + n/25 + n/125 ... (whole parts only).
`Same remainder r for all divisors`	Number = k × LCM + r.

Shortcuts

Greatest / smallest fraction without decimals Use when: Four fractions in the options.

1. Cross-multiply two at a time: a/b vs c/d → compare a*d with b*c.
2. If numerators rise by the same step and denominators by the same step, compare that step fraction to the first: if step fraction is bigger, the list is rising.

Try it: Largest of 3/5, 5/8, 7/11, 9/14

1. Numerators +2, denominators +3. Step fraction 2/3 = 0.67.
2. 2/3 is bigger than 3/5, so each next fraction is bigger. Last one wins.

Answer: 9/14

Remainder of a huge power Use when: 2^{100} / 7 style.

1. Find the smallest power leaving remainder 1 ($2^3 = 8$ → remainder 1).
2. Write the exponent as multiples of that plus leftover: $2^{100} = (2^3)^{33} \times 2$.
3. $(1)^{33} \times 2 = 2$.

Try it: Remainder of 2^{100} / 7

1. 2^3 leaves 1.
2. 2^{99} leaves 1, times 2.

Answer: 2

Common mistakes

Wrong: Calling 1 a prime.	**Instead:** A prime has exactly two factors. 1 has one.
Wrong: Using HCF × LCM = product for three numbers.	**Instead:** It works for two numbers only.
Wrong: Unit digit cycle: using the power itself instead of its remainder by 4.	**Instead:** 7^{95} → 95 mod 4 = 3 → third in the cycle.

Check yourself

- You can test a number for 3, 4, 8, 9 and 11 without dividing.
- You can find the unit digit of a big power from its cycle.
- You can count the factors of a number from its prime powers.
- You can choose HCF or LCM from the words in the question.
- You can find the remainder of a big power.

Class plan (2 hours)

`10 min`	Speed maths warm-up.
`10 min`	Kinds of numbers; test 221 and 391 for primes together.
`20 min`	Divisibility table; find-the-missing-digit questions in pairs.
`15 min`	Unit digits: the cycles on the board.
`15 min`	Factors and the factor tree; number of factors.

`20 min`	HCF and LCM: word clues, bells, tiles, remainder types.
`10 min`	Remainders of products and powers.
`5 min`	Trailing zeros.
`10 min`	Practice set, timed.
`5 min`	Recap and exit ticket: unit digit of 3^{47} (answer 7).

Practice: 19 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/number-system

Percentage

> **Percent means 'per hundred'. 30% is 30 out of every 100. That is the whole topic; everything else is choosing which number is the 100.**

Percentage is inside profit and loss, discount, interest, population, elections and every DI chart. The one skill that decides most answers: knowing what the percentage is OF. The base changes from question to question.

What a percentage is, and finding x% of a number

x% of a number = x/100 × the number. 30% of 250 = 30/100 × 250 = 75.

Quicker: 10% is one tenth, 1% is one hundredth. 30% of 250 = 3 × 25 = 75. 15% = 10% + 5% (half of 10%).

To write a number as a percentage of another: (part / whole) × 100. 45 out of 60 = 45/60 × 100 = 75%.

30% of 250: split the whole into 100 equal parts, take 30

Example 1: What is 35% of 640?

1. 10% of 640 = 64.
   Why: Move the decimal point one place.
2. 30% = 3 × 64 = 192. 5% = half of 64 = 32.
   Why: Build 35% from easy pieces.
3. 192 + 32 = 224.

Answer: 224

Example 2: A student scores 234 out of 360. What percentage is that?

1. 234/360 × 100.
   Why: Part over whole, times 100.
2. 234/360 = 13/20 = 0.65.
   Why: Divide both by 18.
3. 65%.

Answer: 65%

Increase, decrease, and the multiplier

A 20% increase means the new value is 120% of the old one: multiply by 1.2. A 20% decrease means 80%: multiply by 0.8.

Percentage change = (change / ORIGINAL value) x 100. The original is always the base.

Going back: if a price after a 25% rise is 500, the original is 500 / 1.25 = 400, not 500 - 25% of 500.

`New = old × (1 + r/100)`	Increase by r%.
`New = old × (1 - r/100)`	Decrease by r%.
`% change = (new - old)/old × 100`	Always divide by the old value.

Example 1: After a 25% increase a price is Rs 750. What was it before?

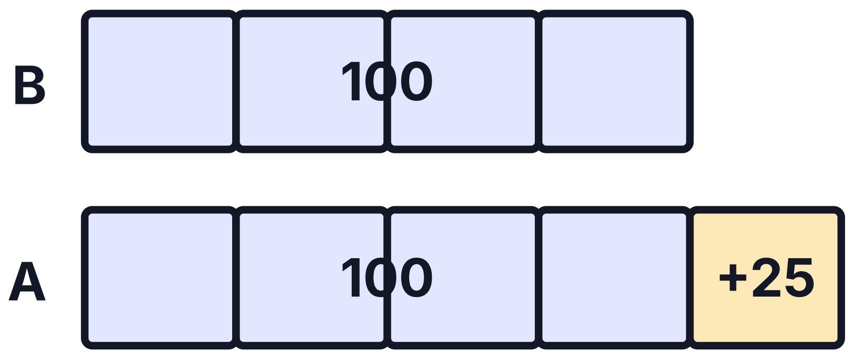
1. New = old × 1.25.
   Why: A 25% increase is 125% of the old price.
2. Old = 750 / 1.25 = 600.
   Why: Divide to go back; do not subtract 25% of 750.

Answer: 600

'More than' and 'less than' are not symmetric

If A is 25% more than B, B is NOT 25% less than A. Take B = 100, then A = 125. B is 25 less than A, and 25/125 = 20%.

Rule: if A is r% more than B, then B is r/(100 + r) × 100 % less than A. If A is r% less than B, B is r/(100 - r) × 100 % more than A.



A = 125 when B = 100. The same 25 is 25% of B but only 20% of A

Example 1: A's income is 20% less than B's. By what percent is B's income more than A's?

1. Let B = 100. Then A = 80.
   Why: B is the base in the sentence.
2. B is 20 more than A.
3. As a percentage of A: 20/80 × 100 = 25%.
   Why: 'More than A' means A is the base now.

Answer: 25%

Successive changes

Two changes one after the other: the second applies to the NEW value, so you cannot just add them.

Net change for a% then b% = a + b + ab/100 (use minus for decreases). +10% then +10% = 21%. +20% then -20% = -4%.

Population questions are successive changes over years: $P \times (1 + r/100)^n$.

Example 1: A town's population of 50,000 grows 10% in the first year and falls 10% in the second. Population after two years?

1. After year 1: 50000 × 1.1 = 55,000.
   Why: Increase applies to the starting population.
2. After year 2: 55000 × 0.9 = 49,500.
   Why: The fall applies to 55,000, not 50,000.
3. 49,500 (a 1% net fall: 10 - 10 - 100/100 = -1).

Answer: 49500

Price and consumption, income and spending

Spending = price × quantity. If the price rises and you want to spend the same, you must use less.

If price rises by r%, consumption must fall by r/(100 + r) × 100 % to keep spending the same. For a 25% rise: 25/125 = 20% cut.

Same idea for income, expenditure and savings: savings = income - expenditure. Take income = 100 and follow each change.

Example 1: The price of sugar rises by 25%. By how much must a family cut consumption to keep spending the same?

1. Take price 100, quantity 100, spend 10,000.
   Why: Take 100 for everything.
2. New price 125. To spend 10,000: quantity = 10000/125 = 80.
3. Cut = 20%.

Answer: 20%

Elections, marks and passing

Election questions: votes for the winner minus votes for the loser = winning margin. If 20% did not vote, only 80% of the voters are counted.

Pass-mark questions: 'A gets 30% and fails by 15 marks, B gets 40% and gets 35 more than the pass mark'. The difference between them, 10% of the maximum, is 15 + 35 = 50 marks. So the maximum is 500.

Example 1: In an election between two candidates, the winner got 58% of the valid votes and won by 1,120 votes. How many valid votes were there?

1. Loser got 42%. Margin = 58 - 42 = 16% of valid votes.
   Why: The margin is the gap between their shares.
2. 16% = 1120, so 1% = 70.
3. Valid votes = 7,000.

Answer: 7000

Example 2: A student needs 40% to pass. He gets 178 marks and fails by 22 marks. What is the maximum mark?

1. Pass mark = 178 + 22 = 200.
   Why: Failing by 22 means 22 short of the pass mark.
2. 40% of max = 200.
3. Max = 500.

Answer: 500

Formula summary

`x% of y`	x/100 × y. Use the fraction table: 12.5% = 1/8, 16.67% = 1/6.
`A is r% more than B → B is less than A by`	r / (100 + r) × 100. (25% more → 20% less.)
`A is r% less than B → B is more than A by`	r / (100 - r) × 100. (20% less → 25% more.)
`Successive changes a% then b%`	a + b + ab/100 (use minus signs for decreases).
`Price up r%, keep spend same`	Cut consumption by r / (100 + r) × 100.
`Population / depreciation`	$P(1 + r/100)^n$, or $(1 - r/100)^n$ for decline.
`Election, 2 candidates`	Winning margin % of valid votes = winner% - loser%.

Shortcuts

Successive change in one line Use when: Any two changes stacked on each other.

1. Net = a + b + ab/100.
2. +20 then -20: 20 - 20 - 400/100 = -4%.

Try it: Price raised 25% then cut 20%. Net change?

1. 25 - 20 - 500/100 = 0.

Answer: 0%

Pass/fail marks pair Use when: 'Scores x% and fails by m; scores y% and gets n more than pass'.

1. The gap in % covers the gap in marks: (y - x)% of Max = m + n.

Try it: 30% fails by 30; 40% passes by 30. Max marks?

1. 10% = 60.
2. Max = 600.

Answer: 600

Common mistakes

Wrong: Undoing a 25% rise by taking 25% off the new price.	**Instead:** Divide by 1.25.
Wrong: Adding successive changes: +10% and +10% = +20%.	**Instead:** It is 21%: the second 10% is on 110.
Wrong: Using the wrong base: 'B is less than A by ...' divided by B.	**Instead:** Divide by the value after 'than'.

Check yourself

- You can find any percentage using 10% and 1% pieces.
- You can go back from a new value by dividing by the multiplier.
- You can explain why 25% more is not 25% less.
- You can combine two changes with a + b + ab/100.
- You can solve election and pass-mark questions.

Class plan (2 hours)

`10 min`	Speed warm-up: fraction-percent table.
`15 min`	What % means; x% of y with 10% and 1% pieces.
`15 min`	Increase, decrease, multiplier, going back.
`15 min`	More than / less than asymmetry with the bar picture.
`15 min`	Successive change and population.
`15 min`	Price and consumption, income and savings.
`15 min`	Elections and marks.

`15 min`	Practice set, timed.
`5 min`	Recap.

Practice: 16 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/percentage

Profit and Loss

> **A shopkeeper buys a shirt for Rs 400 and sells it for Rs 500. The Rs 100 extra is profit, and it is 25% because profit is always compared with what he PAID.**

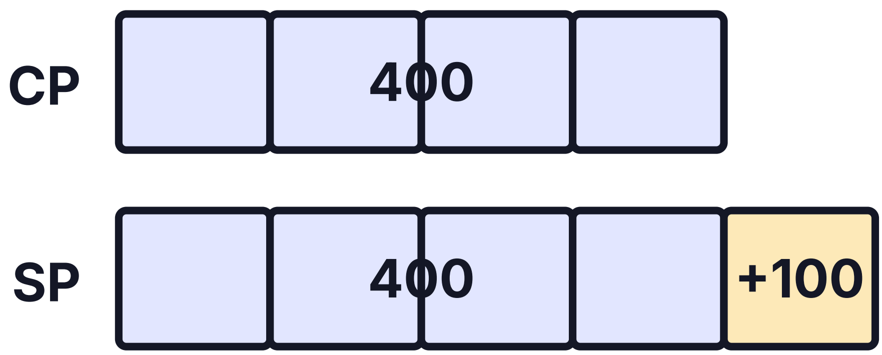
Profit and loss has 1 or 2 questions in nearly every paper. Almost every mistake comes from one thing: taking the percentage on the selling price instead of the cost price.

CP, SP, profit and loss

Cost price (CP): what the seller paid. Selling price (SP): what the buyer pays. If SP > CP there is a profit = SP - CP; if SP < CP there is a loss = CP - SP.

Profit % and loss % are ALWAYS calculated on the cost price: profit % = profit / CP × 100.

So SP = CP × (1 + profit%/100), and CP = SP / (1 + profit%/100).



CP 400, SP 500: profit 100 on a base of 400 = 25%

`Profit % = (SP - CP)/CP × 100`	Always over CP.
`SP = CP × (100 + P%)/100`	
`CP = SP × 100/(100 + P%)`	Going back from SP.
`Loss: SP = CP × (100 - L%)/100`	

Example 1: A watch bought for Rs 1,600 is sold for Rs 2,000. Find the profit percentage.

1. Profit = 2000 - 1600 = 400.
2. Profit % = 400/1600 × 100 = 25%.
   Why: Divide by CP, not SP.

Answer: 25%

Example 2: By selling a bag for Rs 690 a trader gains 15%. What did the bag cost him?

1. SP = CP × 1.15.
   Why: A 15% gain means SP is 115% of CP.
2. CP = 690 / 1.15 = 600.

Answer: **600**

Same CP or same SP, and 'loss on both'

If two items are sold at the SAME selling price, one at x% profit and the other at x% loss, the seller always loses overall: loss % = $x^2/100$.

Why: the loss-making item cost more than the profit-making one (you need a higher CP to end up at the same SP after a loss), so the loss is on a bigger base.

Example 1: Two phones are sold for Rs 9,900 each, one at 10% profit and one at 10% loss. Overall gain or loss?

1. CP1 = 9900/1.1 = 9000. CP2 = 9900/0.9 = 11000.
   Why: Go back from SP for each phone.
2. Total CP 20,000, total SP 19,800.
3. Loss 200 on 20,000 = 1% (= $10^2/100$).

Answer: **1% loss**

Dishonest weights and 'error'

A shopkeeper who uses a 900 g weight instead of 1 kg gives 900 g but charges for 1000 g. His gain % = (error)/(true weight given) × 100 = 100/900 × 100 = 11.11%.

If he also marks up or discounts, apply those as successive changes on top.

Example 1: A dealer sells at cost price but uses a weight of 960 g for 1 kg. His gain %?

1. He gives 960 g and is paid for 1000 g.
   Why: The 40 g difference is his gain.
2. Gain % = 40/960 × 100.
   Why: Base is what he actually gives (his cost).
3. = 4.17%.

Answer: **4.17%**

Successive profits and the 'take 100' method

A makes 10% selling to B, B makes 20% selling to C. C pays 100 × 1.1 × 1.2 = 132 for every 100 that A paid.

Whenever the question has only percentages, take CP = 100 and follow the money.

Example 1: A sells to B at 20% profit, B sells to C at 25% profit. C pays Rs 225. What did A pay?

1. A's CP × 1.2 × 1.25 = 225.
   Why: Two successive profits multiply.
2. 1.2 × 1.25 = 1.5.
3. A's CP = 225 / 1.5 = 150.

Answer: 150

Formula summary

`Profit % = (SP - CP)/CP × 100`	Base is CP.
`SP = CP × (100 + P%)/100`	Loss: (100 - L%).
`Same SP, x% gain on one, x% loss on other`	Always a LOSS of $x^2/100$ %.
`CP of a articles = SP of b articles`	Profit % = (a - b)/b × 100.
`False weight (sells at CP)`	Gain % = error / (true - error) × 100. 900 g for 1 kg → 100/900.
`Chain A→B→C`	Final price = CP × (1 + p1)(1 + p2)...

Shortcuts

Chain of sales backwards Use when: 'A sells to B at p%, B to C at q%, C pays Rs X'.

1. Divide X by each multiplier in turn.

Try it: A→B at 20% profit, B→C at 25% profit, C pays 1500. A's CP?

1. 1500 / 1.25 = 1200.
2. 1200 / 1.2 = 1000.

Answer: Rs 1000

Dishonest dealer: loss price but false weight Use when: Two effects together.

1. Price paid for 'a kilo' vs what that weight really cost.
2. Gain = (what he gets)/(what he gave) - 1.

Try it: Sells at 10% loss, uses 800 g for 1 kg. Gain %?

1. Gets 0.9 of CP for goods worth 0.8.
2. 0.9/0.8 = 1.125.

Answer: 12.5%

Common mistakes

Wrong: Profit % over SP.	**Instead:** Profit % is over CP unless the question says otherwise.

Wrong: CP = SP - 15% of SP.	**Instead:** CP = SP / 1.15.

Check yourself

- You can compute profit or loss % on the cost price.
- You can find CP from SP and a percentage.
- You can explain why equal % profit and loss at the same SP is a loss.
- You can solve false-weight questions.

Class plan (2 hours)

`10 min`	Warm-up.
`25 min`	CP, SP, profit %, going back from SP; five quick examples.
`20 min`	Same SP, one profit one loss: the $x^2/100$ loss.
`20 min`	Dishonest weights.
`15 min`	Successive profits, take 100.
`25 min`	Practice set, timed.
`5 min`	Recap.

Practice: 11 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/profit-and-loss

Discount

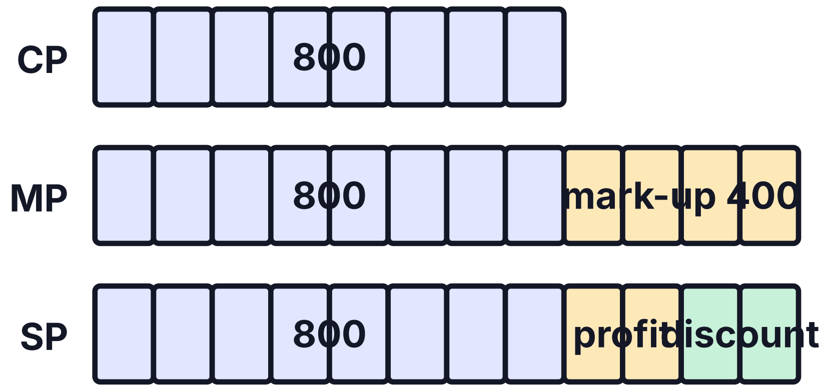
> **The price on the tag is the marked price. The discount comes off the marked price. Profit is still measured against what the shop paid.**

Discount questions combine two percentages on two different bases: discount on the marked price, profit on the cost price. Keep the three prices apart and the rest is arithmetic.

MP, SP and discount

Marked price (MP) or list price: the tag. Discount = MP - SP. Discount % is always on the MP: discount % = discount / MP × 100.

SP = MP × (1 - d/100).



CP 800, marked up to 1200, sold at 1000: discount on MP, profit on CP

Example 1: A jacket marked Rs 2,400 is sold at a 15% discount. Selling price?

1. Discount = 15% of 2400 = 360.
   Why: Discount is on the marked price.
2. SP = 2400 - 360 = 2040.

Answer: 2040

Successive discounts

'20% + 10% off' is not 30%. The 10% is taken on the price after the first discount.

Single equivalent discount for a% and b% = a + b - ab/100. 20% and 10% → 30 - 2 = 28%.

Example 1: Find the single discount equal to successive discounts of 20% and 15%.

1. a + b - ab/100 = 20 + 15 - 300/100.
   Why: The second discount is on the reduced price.
2. = 32%.

Answer: 32%

Marked price, discount and profit together

Take CP = 100. Mark up by m%: MP = 100 + m. Give d% discount: SP = MP × (1 - d/100). Profit % = SP - 100.

Or backwards: to make p% profit after a d% discount, mark up so that MP × (1 - d/100) = CP × (1 + p/100).

Example 1: A shopkeeper marks goods 40% above cost and gives a 20% discount. Profit %?

1. CP = 100, MP = 140.
   Why: Take 100.
2. SP = 140 × 0.8 = 112.
   Why: Discount on MP.
3. Profit 12%.

Answer: 12%

Example 2: At what % above cost should an item be marked so that after a 10% discount the shop still makes 17% profit?

1. CP = 100, needed SP = 117.
   Why: Profit on CP.
2. MP × 0.9 = 117, so MP = 130.
   Why: SP after discount must be 117.
3. Mark up 30%.

Answer: 30%

'Buy × get y free'

Buy 4 get 1 free means you pay for 4 and take 5. Discount = 1/5 = 20% (the free item over the total you take).

Example 1: A shop offers 'buy 3 get 2 free'. What is the effective discount?

1. You take 5 and pay for 3.
2. Free 2 out of 5.
   Why: Discount is over what you receive.
3. 40%.

Answer: 40%

Formula summary

`SP = MP × (100 - d)/100`	Discount is on MP.
`Successive discounts a%, b%`	Single equivalent = a + b - ab/100.
`MP / CP = (100 + P%)/(100 - D%)`	The one link between marked price and profit.
`Buy × get y free`	Discount % = y/(x + y) × 100.

Shortcuts

Markup needed for a target profit Use when: 'How much above cost should he mark?'

1. MP/CP = (100 + P)/(100 - D).

Try it: Wants 20% profit after a 25% discount. Mark up by?

1. 120/75 = 1.6.

Answer: 60%

Common mistakes

Wrong: 20% and 10% successive = 30%.	**Instead:** 28%: a + b - ab/100.
Wrong: Profit % on the marked price.	**Instead:** Profit is always on the cost price.

Check yourself

- You can take discount on MP and profit on CP.
- You can combine two discounts with a + b - ab/100.
- You can find the mark-up needed for a target profit.

Class plan (2 hours)

`10 min`	Warm-up.
`20 min`	Three prices: CP, MP, SP on the bar picture.
`20 min`	Successive discounts.
`30 min`	Mark-up with discount, both directions.
`10 min`	Buy × get y free.
`25 min`	Practice set.
`5 min`	Recap.

Practice: 10 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/discount

Ratio and Proportion

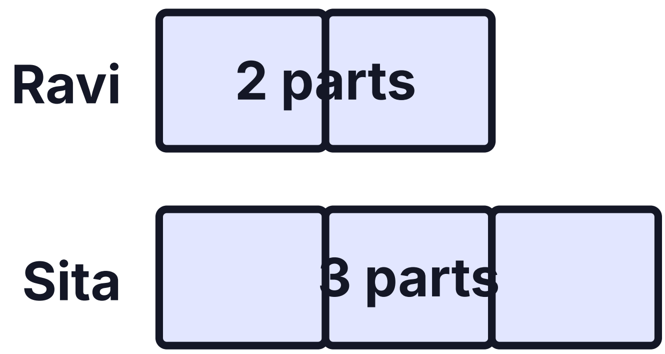
> **If a recipe uses 2 cups of rice for every 3 cups of water, the ratio is 2 : 3. The actual amounts can be 4 and 6 or 10 and 15. A ratio is a pattern of parts.**

Ratios drive partnership, ages, mixtures, time and work and DI. The single best tool is the bar model: draw the parts as equal boxes and find what one box is worth.

What a ratio means: parts

a : b means for every a parts of the first there are b parts of the second. Multiply or divide both by the same number and the ratio is unchanged: 4 : 6 = 2 : 3.

To split an amount in the ratio a : b, the total is a + b parts. One part = amount / (a + b).



Rs 1,000 in the ratio 2 : 3 is 5 parts, so one part = 200

Example 1: Divide Rs 1,000 between Ravi and Sita in the ratio 2 : 3.

1. Total parts = 2 + 3 = 5.
   Why: Each box in the bar model is one part.
2. One part = 1000 / 5 = 200.
3. Ravi 400, Sita 600.

Answer: 400

Combining ratios

If A : B = 2 : 3 and B : C = 4 : 5, make B the same in both. LCM of 3 and 4 is 12: A : B = 8 : 12 and B : C = 12 : 15, so A : B : C = 8 : 12 : 15.

Compound ratio of a : b and c : d is ac : bd.

Example 1: A : B = 3 : 4 and B : C = 6 : 7. Find A : C.

1. Make B equal: LCM(4, 6) = 12.
   Why: B must be the same number of parts in both ratios.
2. A : B = 9 : 12, B : C = 12 : 14.
3. A : C = 9 : 14.

Answer: 9 : 14

Changing a ratio by adding or removing

When something is added to both terms, the difference between them does not change. Use that: it is the easiest thing to keep track of.

Set up the new ratio and solve for the unknown, or try the options.

Example 1: Two numbers are in the ratio 3 : 5. If 9 is added to each, the ratio becomes 3 : 4. Find the numbers.

1. Numbers 3k and 5k. $(3k + 9)/(5k + 9) = 3/4$.
2. $4(3k + 9) = 3(5k + 9) \rightarrow 12k + 36 = 15k + 27 \rightarrow k = 3$.
   Why: Cross-multiply.
3. Numbers 9 and 15.

Answer: 9 and 15

Proportion: direct and inverse

Four numbers are in proportion if a : b = c : d, i.e. $a \times d = b \times c$ (product of extremes = product of means).

Direct proportion: both go up together (more items, more cost). Inverse proportion: one goes up as the other goes down (more workers, fewer days).

Mean proportional between a and b is $\sqrt{(ab)}$. Third proportional to a and b is b^2/a. Fourth proportional to a, b, c is bc/a.

Example 1: Find the fourth proportional to 4, 9 and 12.

1. 4 : 9 = 12 : x.
   Why: The fourth term completes the proportion.
2. $4x = 9 \times 12 = 108$.
   Why: Extremes = means.
3. x = 27.

Answer: 27

Example 2: Find the mean proportional between 9 and 16.

1. √(9 × 16) = √(144).
2. = 12.

Answer: 12

Coins, ages and other ratio stories

A bag has 1-rupee, 50-paise and 25-paise coins in the ratio 5 : 6 : 8 and a total value of Rs 210. Value ratio = 5 × 1 : 6 × 0.5 : 8 × 0.25 = 5 : 3 : 2. So the value splits 105 : 63 : 42 and the 1-rupee coins number 105.

Example 1: A bag has 1-rupee, 50-paise and 25-paise coins in the ratio 5 : 6 : 8, worth Rs 210 in all. How many 1-rupee coins?

1. Value per 'set' of coins: 5 × 1 + 6 × 0.5 + 8 × 0.25 = 5 + 3 + 2 = 10 rupees.
   Why: Turn the count ratio into money.
2. Number of sets = 210 / 10 = 21.
3. 1-rupee coins = 5 × 21 = 105.

Answer: 105

Formula summary

`Combine a:b and b:c`	Make the common term equal: multiply across.
`Mean proportional of a, b`	√(ab).
`Third proportional of a, b`	b^2 / a.
`Fourth proportional of a, b, c`	bc / a.
`Divide in 1/a : 1/b : 1/c`	Multiply by the LCM of a, b, c to get whole numbers first.

Shortcuts

Coins by value Use when: Bag of coins given in ratio by COUNT, total given in rupees.

1. Convert count ratio to value ratio: multiply each by its coin value.
2. Share the total by value ratio, then divide by coin value.

Try it: Rs 1, 50p, 25p coins in count ratio 5:6:8, total Rs 210. How many 50p coins?

1. Value ratio 5 : 3 : 2 = 10 parts, each Rs 21.
2. 50p value = 63 → 126 coins.

Answer: 126

Common mistakes

Wrong: Adding the same number to both terms and keeping the ratio.	**Instead:** Adding changes the ratio; multiplying does not.
Wrong: Combining A : B and B : C without making B equal.	**Instead:** Scale both ratios so B has the same number.

Check yourself

- You can split an amount in a given ratio with the bar model.
- You can combine two ratios through the common term.
- You can find mean, third and fourth proportionals.

Class plan (2 hours)

`10 min`	Warm-up.
`20 min`	Ratio as parts with the bar model; splitting money.
`15 min`	Combining ratios.
`20 min`	Adding to both terms.
`20 min`	Proportion: direct, inverse, mean/third/fourth proportional.
`10 min`	Coins.
`20 min`	Practice set.
`5 min`	Recap.

Practice: 9 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/ratio-and-proportion

Partnership

> **Two friends start a shop. One puts in Rs 30,000, the other Rs 20,000. If the shop earns Rs 10,000, it is fair to split it 3 : 2. Partnership is just that fairness rule.**

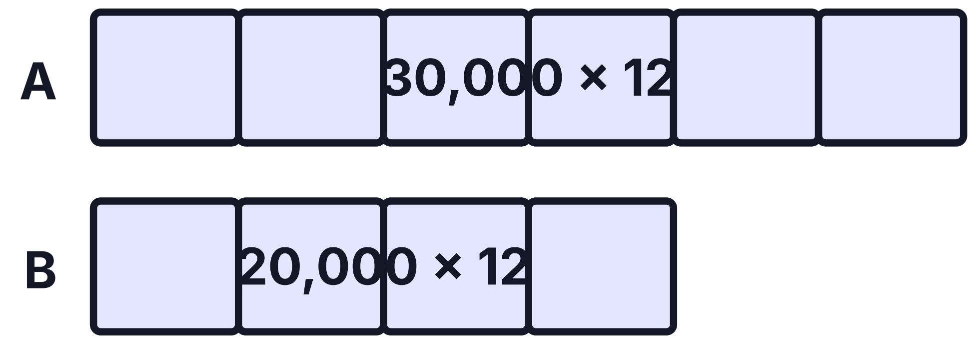
Partnership is ratio with one extra idea: money that stays in the business longer counts for more. Profit is shared in the ratio of (money × time).

Profit share = capital × time

If everyone invests for the same time, share profit in the ratio of the money invested.

If the times differ, multiply each person's money by the number of months it stayed in. Rs 10,000 for 12 months counts the same as Rs 20,000 for 6 months.

A sleeping partner only invests; a working partner also runs the business and may get a salary or a fixed % of profit first. Take that out before splitting the rest.



Same time: profit 3 : 2

`Share ratio = C1 × T1 : C2 × T2 : ...`	Capital times months.
`Working partner: salary first, then split the rest`	Read carefully what is paid before the split.

Example 1: A invests Rs 40,000 for a year. B joins after 4 months with Rs 60,000. The year's profit is Rs 34,000. B's share?

1. A: 40,000 × 12 = 4,80,000. B: 60,000 × 8 = 4,80,000.
   Why: B's money was in for 12 - 4 = 8 months.
2. Ratio 1 : 1.
   Why: Equal capital-months.
3. B gets 17,000.

Answer: 17000

Example 2: A and B start with Rs 50,000 and Rs 30,000. After 6 months A withdraws Rs 20,000. Profit at year end is Rs 23,000. Find A's share.

1. A: 50,000 × 6 + 30,000 × 6 = 4,80,000.
   Why: Split A's year into the two periods with different amounts.
2. B: 30,000 × 12 = 3,60,000.
3. Ratio 480 : 360 = 4 : 3.
   Why: Divide by 1,20,000.
4. A = 4/7 × 23,000 = 13,142.86.

Answer: 13,142.86

Formula summary

`Share ratio`	C1 × T1 : C2 × T2 : ...
`Joins late / leaves early`	Count only the months that money was actually in.
`Withdraw or add mid-year`	Split that partner into two blocks: (old capital × months) + (new capital × months).
`Working partner`	Take out the salary / commission first, split the rest by capital ratio.
`Given profit ratio and time ratio`	Capital ratio = profit / time for each.

Shortcuts

Rupee-months table Use when: Any partnership with changes.

1. One row per partner, sum of capital × months.
2. Reduce the totals to a ratio before dividing profit.

Try it: A: 40,000 all year but withdraws 10,000 after 6 months. B: 60,000 all year. Profit 38,000. A's share?

1. A = 40k × 6 + 30k × 6 = 4.2 lakh. B = 7.2 lakh.
2. 7 : 12, 19 parts of 2000.
3. A = 14,000.

Answer: Rs 14,000

Common mistakes

Wrong: Using the money ratio when the partners were in for different months.	**Instead:** Multiply each amount by its months.

Check yourself

- You can share profit in the ratio of capital × months.

- You can handle a partner joining late or withdrawing money midway.
- You can take out a working partner's salary first.

Class plan (2 hours)

`10 min`	Warm-up.
`25 min`	Equal time: profit in the money ratio.
`30 min`	Different times; joining and leaving.
`15 min`	Adding or withdrawing midway.
`10 min`	Working partner salary.
`25 min`	Practice set.
`5 min`	Recap.

Practice: 7 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/partnership

Simple Interest

> **You lend Rs 1,000 at 10% a year. Every year you get Rs 100, the same Rs 100, because simple interest is always on the original amount.**

Simple interest is one formula, SI = P R T / 100, used in four directions: find the interest, the principal, the rate or the time.

The formula and what each letter means

P is the principal (the money lent). R is the rate per year in %. T is the time in years. SI = P × R × T / 100.

Amount A = P + SI: what you get back at the end.

Time in months? Divide by 12. 9 months = 3/4 year.

`SI = P R T / 100`	
`A = P + SI = P (1 + RT/100)`	
`P = 100 SI / (R T)`	Rearranged.
`R = 100 SI / (P T)`	

Example 1: Find the simple interest on Rs 8,000 at 7.5% a year for 3 years.

1. SI = 8000 × 7.5 × 3 / 100.
   Why: Put the numbers in.
2. = 80 × 22.5 = 1,800.

Answer: 1800

Example 2: At what rate will Rs 2,500 earn Rs 600 simple interest in 4 years?

1. R = 100 × 600 / (2500 × 4).
   Why: Rearranged formula.
2. = 60000 / 10000 = 6%.

Answer: 6%

Doubling and tripling

If a sum doubles, the interest equals the principal: SI = P, so R × T = 100.

If it becomes n times, SI = (n - 1) P, so R × T = 100 (n - 1). A sum that doubles in 8 years (R = 12.5%) becomes 4 times in 24 years, not 16.

Example 1: A sum doubles in 8 years at simple interest. In how many years will it become 5 times?

1. Doubling: interest = P in 8 years, so R = 12.5%.
   Why: R × 8 = 100.
2. 5 times: interest = 4P, so R × T = 400.
   Why: n - 1 = 4.
3. T = 400 / 12.5 = 32 years.

Answer: 32

Two amounts, two times

'A sum amounts to Rs 1,120 in 2 years and Rs 1,200 in 3 years.' The extra Rs 80 is one year's interest. Two years' interest = 160, so P = 1120 - 160 = 960.

Example 1: A sum amounts to Rs 7,000 in 4 years and Rs 8,500 in 7 years at simple interest. Find the principal and the rate.

1. 3 extra years give 1,500, so one year = 500.
   Why: Simple interest is the same every year.
2. 4 years = 2,000, so P = 7000 - 2000 = 5,000.
3. R = 500/5000 × 100 = 10%.

Answer: P = Rs 5,000 and R = 10%

Formula summary

`SI = P × R × T / 100`	The one formula.
`Amount = P + SI`	
`Doubles in T years`	R = 100/T. Becomes n times in (n - 1) × 100 / R years.
`Two amounts at two times`	Yearly SI = difference / year gap. P = earlier amount - years × yearly SI.

Shortcuts

Two totals, two times Use when: 'Becomes 7,200 in 3 years and 8,400 in 5 years'.

1. 2 years of interest = 1200 → 600 a year.
2. P = 7200 - 3 × 600.

Try it: As above. Principal?

1. 7200 - 1800.

Answer: Rs 5,400

Check yourself

- You can use SI = PRT/100 to find any one letter.

- You can solve doubling and n-times questions with RT = 100(n - 1).
- You can get P and R from two amounts at two times.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	The formula in four directions.
`25 min`	Doubling, tripling.
`20 min`	Two amounts at two times.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 6 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/simple-interest

Compound Interest

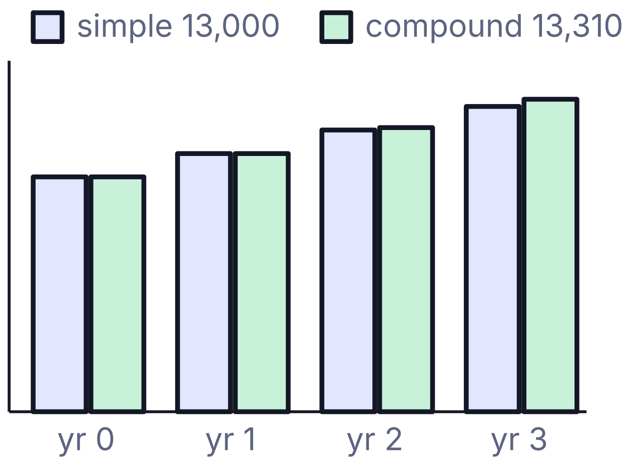
> **With compound interest, last year's interest starts earning interest too. Rs 1,000 at 10% becomes 1,100, then 1,210, then 1,331.**

Compound interest questions test the multiplier $(1 + R/100)^n$, the difference between CI and SI, and half-yearly or quarterly compounding.

Interest on interest

Amount after n years: $A = P(1 + R/100)^n$. CI = A - P.

Year by year it is successive percentage increases: each year multiplies by (1 + R/100).



Rs 10000 at 10%: compound pulls ahead because interest earns interest

$A = P(1 + R/100)^n$	
`CI = A - P`	
`Half-yearly: rate R/2, periods 2n`	Quarterly: R/4 and 4n.

Example 1: Find the compound interest on Rs 10,000 at 10% a year for 3 years.

1. $A = 10000 \times 1.1^3$.
   Why: Multiply by 1.1 each year.
2. $1.1^3 = 1.331$, so A = 13,310.
3. CI = 3,310.

Answer: 3310

Example 2: Rs 8,000 at 10% a year compounded half-yearly for 1 year. Amount?

1. Half-yearly: rate 5% per half-year, 2 periods.
   Why: Halve the rate, double the periods.
2. $A = 8000 \times 1.05^2 = 8000 \times 1.1025 = 8{,}820$.

Answer: 8820

CI minus SI

For 2 years: CI - SI = P $(R/100)^2$. That is the 'interest on the first year's interest'.

For 3 years: CI - SI = P $(R/100)^2$ (3 + R/100).

Example 1: The difference between CI and SI on a sum at 10% for 2 years is Rs 50. Find the sum.

1. CI - SI = P × $(10/100)^2$ = P/100.
   Why: Two-year formula.
2. P/100 = 50, so P = 5,000.

Answer: 5000

Doubling and the rule of 72

At compound interest, if a sum doubles in n years it becomes 4 times in 2n, 8 times in 3n. Powers, not multiples.

Rough rule: money doubles in about 72/R years. At 8%, about 9 years.

Example 1: A sum doubles in 5 years at compound interest. In how many years will it be 8 times?

1. 8 = 2^3.
   Why: Three doublings.
2. 3 × 5 = 15 years.

Answer: 15

Formula summary

$A = P(1 + R/100)^n$	
Half-yearly	Rate R/2, periods 2n. Quarterly: R/4 and 4n. Monthly: R/12 and 12n.
CI - SI, 2 years	$P(R/100)^2$.
CI - SI, 3 years	P R^2 (300 + R) / 100^3.
Doubles in n years	Becomes 4x in 2n, 8x in 3n.
Two amounts, consecutive years	Rate = (later - earlier)/earlier × 100.

Shortcuts

10% CI by hand Use when: Build the year-by-year amounts; it is faster than powers.

1. Year 1: add 10%. Year 2: add 10% of the new amount.

Try it: CI on 10,000 at 10% for 3 years

1. 11000, 12100, 13310.
2. CI = 3310.

Answer: Rs 3,310

Common mistakes

Wrong: Doubling in 5 years means 4 times in 10 years at SI.	**Instead:** Only at CI. At SI, 4 times takes 15 years.

Check yourself

- You can compute CI with the multiplier.
- You can adjust rate and periods for half-yearly and quarterly.
- You can use CI - SI = $P(R/100)^2$ for two years.

Class plan (2 hours)

`10 min`	Warm-up.
`15 min`	Year-by-year table vs SI (the chart).
`25 min`	$A = P(1 + R/100)^n$, CI.
`20 min`	Half-yearly and quarterly.
`20 min`	CI - SI for 2 and 3 years.
`10 min`	Doubling powers; rule of 72.
`15 min`	Practice set.
`5 min`	Recap.

Practice: 11 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/compound-interest

Time and Work

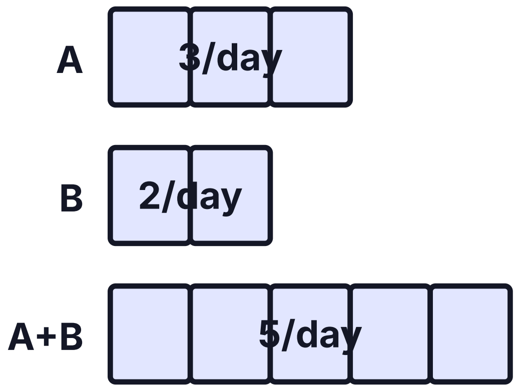
> **If A paints a wall in 10 days, then in one day A paints one tenth of it. Time and work is about per-day work, because per-day work can be added.**

2 or 3 questions in most papers. Days cannot be added; work per day can. The LCM method from Easy Maths turns every question into whole numbers.

Work per day, and the LCM total

If A takes a days, A does 1/a of the job per day. Working together, rates add: 1/a + 1/b.

Faster: let the total work be the LCM of the days. Then each person does a whole number of units per day.



Job = 30 units (LCM of 10 and 15). Together 5 a day: 6 days

`Together: T = ab/(a + b)`	Two people only.
`Total work = LCM of the days`	Units per day = total / days.
`M1 D1 H1 / W1 = M2 D2 H2 / W2`	Men × days × hours per unit of work stays the same.

Example 1: A can do a job in 12 days, B in 18 days. They work together for 4 days, then A leaves. How long does B take to finish?

1. Total = LCM(12, 18) = 36 units. A does 3/day, B does 2/day.
   Why: Whole numbers, no fractions.
2. 4 days together: 4 × 5 = 20 units done.
3. Left: 16 units. B alone: 16 / 2 = 8 days.
4. 8 more days.

Answer: 8

Efficiency

'A is twice as efficient as B' means A does twice the work per day, so A takes half the time.

If A is x% more efficient than B, A's rate = B's rate × (1 + x/100), so A's time = B's time / (1 + x/100).

Example 1: A is twice as efficient as B. Together they finish a job in 14 days. How long would A take alone?

1. Rates 2 : 1. Together 3 units a day.
   Why: Efficiency is work per day.
2. Total = 3 × 14 = 42 units.
3. A alone: 42 / 2 = 21 days.

Answer: 21

Men, days and hours

The amount of work = number of workers × days × hours per day. If the job is the same, that product is the same.

If the job changes, divide by the work: M1 D1 H1 / W1 = M2 D2 H2 / W2.

Example 1: 12 men working 8 hours a day finish a job in 10 days. How many days will 16 men working 6 hours a day take?

1. 12 × 8 × 10 = 16 × 6 × D.
   Why: Same job, so the same total man-hours.
2. 960 = 96 D, so D = 10.

Answer: 10

Alternate days

If A and B work on alternate days, treat one pair of days as a block. Work in a block = A's rate + B's rate.

Fill whole blocks, then finish the remainder carefully day by day (whoever's turn it is).

Example 1: A can do a job in 10 days and B in 15. They work on alternate days, starting with A. In how many days is the job done?

1. Total 30 units. A 3/day, B 2/day. One 2-day block = 5 units.
2. 5 blocks = 10 days = 25 units. Left 5.
3. Day 11 (A): 3 units, left 2. Day 12 (B): 2 units, done.
   Why: Check whose turn it is after the blocks.
4. 12 days.

Answer: 12

Formula summary

`Together`	1/a + 1/b = 1/T → T = ab/(a + b).
`LCM method`	Total work = LCM of days. Each person's rate = LCM / their days.

`Pairs given (A+B, B+C, C+A)`	Add all three rates and halve to get A+B+C.
`Men-days-hours`	M1 D1 H1 / W1 = M2 D2 H2 / W2.
`One leaves × days before the end`	Let total be T; the leaver worked T - x days.

Shortcuts

LCM method Use when: Three or more people, or pairs.

1. A 10, B 15, C 30 → total 30 units.
2. Rates 3, 2, 1 = 6 a day.
3. 30/6 = 5 days.

Try it: A 10 days, B 15, C 30 together?

1. 30 units, 6 per day.

Answer: 5 days

Alternate days Use when: A and B work on alternate days.

1. Units per 2-day cycle.
2. Run full cycles, then finish the leftover with whoever is next.

Try it: A 12 days, B 18 days, alternate, A starts.

1. 36 units. A 3, B 2 → 5 per cycle.
2. 7 cycles = 14 days, 35 units.
3. Day 15: A needs 1 unit = 1/3 day.

Answer: 14 1/3 days

Common mistakes

Wrong: A takes 10 days and B 15, so together 25 (or 12.5).	**Instead:** Add rates: 1/10 + 1/15 = 1/6, so 6 days.

Check yourself

- You can add rates, not days.
- You can set the job to the LCM of the days.
- You can use M × D × H = constant.
- You can solve alternate-day questions with blocks.

Class plan (2 hours)

`10 min`	Warm-up.
`25 min`	Rates and the LCM total; together; one leaves.

`20 min`	Efficiency.
`20 min`	Men-days-hours.
`15 min`	Alternate days.
`25 min`	Practice set.
`5 min`	Recap.

Practice: 11 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/time-and-work

Pipes and Cisterns

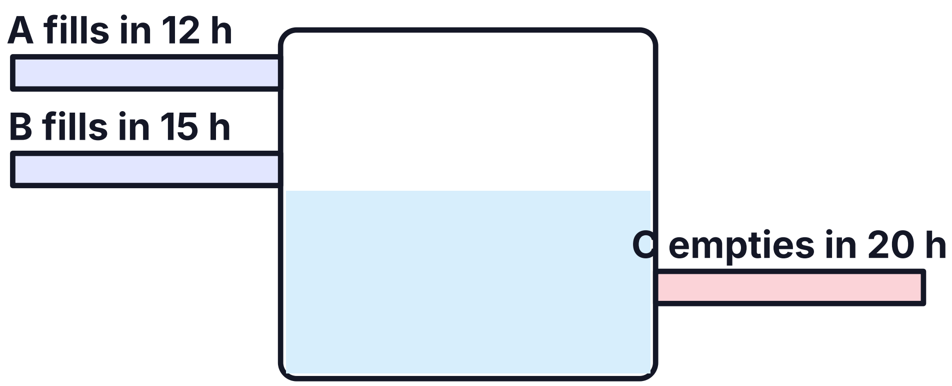
> **A pipe that fills a tank is a worker. A pipe that empties it is a worker doing negative work. That is the only difference from time and work.**

Same method as time and work: total = LCM, inlets add, outlets subtract.

Inlets add, outlets subtract

Let the tank's capacity be the LCM of the times. Each filling pipe adds its units per hour, each emptying pipe takes away its units per hour.

If the net rate is negative, the tank never fills; it empties.



Tank = 60 units. A +5, B +4, C -3: net +6 an hour, full in 10 h

Example 1: Pipes A and B fill a tank in 12 and 15 hours; C empties it in 20 hours. All three open together: how long to fill?

1. Tank = LCM(12, 15, 20) = 60 units.
2. A +5, B +4, C -3 per hour.
   Why: Emptying counts as negative.
3. Net +6 per hour.
4. 60 / 6 = 10 hours.

Answer: 10

A leak, and a pipe closed partway

A leak: a tank normally fills in 6 hours but takes 8 with a leak. Tank = 24 units: pipe +4, together +3, so the leak is -1 an hour and empties a full tank in 24 hours.

Pipe closed after some time: work out what was filled while it was open, then finish the rest with the pipes still running.

Example 1: A pipe fills a tank in 6 hours. Because of a leak it takes 8 hours. How long would the leak take to empty a full tank?

1. Tank = LCM(6, 8) = 24.
2. Pipe +4/h, with leak +3/h, so leak = -1/h.
   Why: The difference is the leak.
3. 24 hours.

Answer: 24

Example 2: Pipes A and B fill a tank in 20 and 30 minutes. Both are opened, and A is closed after 5 minutes. How long does B take to finish?

1. Tank 60. A 3/min, B 2/min.
2. 5 minutes together: 25 units.
3. Left 35, B alone: 17.5 minutes.

Answer: 17.5

Formula summary

`Net rate`	Sum of 1/fill times - sum of 1/empty times.
`Leak`	1/leak = 1/normal - 1/with-leak.
`One closed after t minutes`	What the closed pipe did in t, plus what the rest do to finish, = 1.

Shortcuts

Leak time Use when: 'Fills in 6 h, takes 8 h because of a leak'.

1. Leak rate = 1/6 - 1/8 = 1/24.

Try it: As above. Leak alone empties in?

1. 24 hours.

Answer: 24 hours

Check yourself

- You can treat emptying pipes as negative rates.
- You can find a leak's rate from the slower filling time.
- You can handle a pipe closed partway.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Inlets and outlets with the tank picture.

`25 min`	Leaks.
`20 min`	Pipe closed after a while.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 7 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/pipes-and-cisterns

Time and Distance

> **Speed is distance per hour. If you drive at 60 km/h for 2 hours, you cover 120 km. Every question is that one line: distance = speed × time.**

Time and distance and its two cousins (trains, boats) make up 2 to 4 questions per paper. Units and relative speed cause most mistakes.

The formula and units

Distance = speed × time. Speed = distance / time. Time = distance / speed.

km/h to m/s: multiply by 5/18. m/s to km/h: multiply by 18/5. 72 km/h = 20 m/s. 36 km/h = 10 m/s.

When distance is fixed, speed and time are inversely proportional: go 1.5 times faster and take 2/3 the time.

`D = S × T`	
`km/h × 5/18 = m/s`	Because 1 km/h = 1000 m / 3600 s.
`Same distance: S1 / S2 = T2 / T1`	Inverse proportion.

Example 1: A car covers 180 km in 2 hours 30 minutes. What is its speed in m/s?

1. Speed = 180 / 2.5 = 72 km/h.
   Why: 2 h 30 min = 2.5 h.
2. 72 × 5/18 = 20 m/s.
   Why: Convert units last.

Answer: 20

Example 2: Walking at 4/5 of his usual speed, a man is 10 minutes late. What is his usual time?

1. Speed × 4/5 means time × 5/4.
   Why: Same distance: time is inversely proportional to speed.
2. The extra 1/4 of the usual time = 10 minutes.
3. Usual time = 40 minutes.

Answer: 40

Average speed

Average speed = total distance / total time. It is NOT the average of the speeds, unless the TIMES are equal.

For equal distances at speeds a and b: average = 2ab/(a + b).

Example 1: A person goes to office at 40 km/h and returns at 60 km/h. Average speed?

1. Same distance both ways, so use 2ab/(a + b).
   Why: He spends longer at the slower speed, so the average is below 50.
2. 2 × 40 × 60 / 100 = 48 km/h.

Answer: 48

Relative speed

Two objects moving towards each other close the gap at the SUM of their speeds. Moving in the same direction, at the DIFFERENCE.

Meeting time = gap / relative speed.

towards each other: relative speed = sum

Opposite directions: speeds add

same direction: relative speed = difference

Same direction: speeds subtract

Example 1: Two towns are 300 km apart. Two cars start towards each other at 70 and 50 km/h. When do they meet?

1. Relative speed = 70 + 50 = 120 km/h.
   Why: Towards each other: add.
2. 300 / 120 = 2.5 hours.

Answer: 2.5

Example 2: A thief runs at 8 km/h. A policeman starts chasing 10 minutes later at 10 km/h from the same point. When does he catch the thief?

1. Head start: 8 × 10/60 = 4/3 km.
   Why: Distance the thief covers before the chase begins.
2. Closing speed = 10 - 8 = 2 km/h.
   Why: Same direction: subtract.
3. Time = (4/3) / 2 = 2/3 h = 40 minutes.

Answer: 40

Formula summary

`km/h to m/s`	x 5/18. m/s to km/h: x 18/5.
`Average speed, equal distances`	2xy/(x + y). Never (x + y)/2.
`Fixed distance`	Speed ratio a:b → time ratio b:a.
`Late/early at two speeds`	d/s1 - d/s2 = total time gap.

`Speed x/y of usual, late by t`	Usual time = t × x/(y - x).
`Relative speed`	Same direction subtract, opposite directions add.

Shortcuts

Usual time from 'fraction of speed' Use when: 'At 3/4 of usual speed he is 20 min late'.

1. Time becomes 4/3 of usual; the extra 1/3 = 20 min.

Try it: As above. Usual time?

1. 1/3 of usual = 20 → 60.

Answer: 60 min

Common mistakes

Wrong: Average of 40 and 60 is 50.	**Instead:** For equal distances it is 2ab/(a + b) = 48.
Wrong: Mixing km/h with metres and seconds.	**Instead:** Convert everything first.

Check yourself

- You can convert between km/h and m/s.
- You can use average speed = total distance / total time.
- You can add speeds for opposite directions, subtract for the same direction.

Class plan (2 hours)

`10 min`	Warm-up: km/h to m/s for 18, 36, 54, 72, 90.
`25 min`	D = S × T, units, inverse proportion (late/early).
`20 min`	Average speed and 2ab/(a+b).
`30 min`	Relative speed: meeting and chasing, with the diagrams.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 7 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/time-and-distance

Problems on Trains

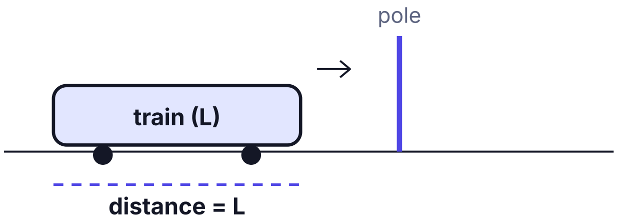
> **A train is not a dot. To pass a pole it must move its whole length. To cross a platform it must move its length plus the platform's.**

Train questions are time and distance with one twist: the train's own length is part of the distance.

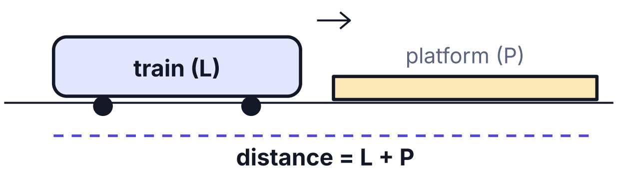
Pole or person vs platform or bridge

Passing a pole, a signal post or a standing person: distance = length of the train.

Crossing a platform, bridge or tunnel: distance = length of the train + length of the platform.



Passing a pole or a person: the train covers its own length



Passing a platform or bridge: its own length plus the platform

Example 1: A train 240 m long passes a pole in 12 seconds. How long will it take to cross a platform 360 m long?

1. Speed = 240 / 12 = 20 m/s.
   Why: Passing a pole: distance = train length.
2. Platform: distance = 240 + 360 = 600 m.
3. 600 / 20 = 30 seconds.

Answer: 30

Two trains

Two trains passing each other cover the SUM of their lengths. Opposite directions: relative speed = sum of speeds. Same direction: difference.

Example 1: Two trains 150 m and 100 m long run on parallel tracks at 54 km/h and 36 km/h in opposite directions. How long to cross each other?

1. Relative speed = 54 + 36 = 90 km/h = 25 m/s.
   Why: Opposite directions add; convert to m/s.
2. Distance = 150 + 100 = 250 m.
   Why: Both lengths.
3. 250 / 25 = 10 seconds.

Answer: 10

Example 2: A 200 m train at 72 km/h overtakes a man walking at 6 km/h in the same direction. How long does it take?

1. Relative speed = 72 - 6 = 66 km/h = 55/3 m/s.
   Why: Same direction subtracts.
2. 200 / (55/3) = 600/55 = 120/11 seconds, about 10.9 s.

Answer: 120/11 s (about 10.9 s)

Formula summary

`Crossing a pole / man`	Time = train length / speed.
`Crossing platform / bridge`	Time = (train + platform) / speed.
`Two trains, opposite`	(L1 + L2)/(s1 + s2).
`Two trains, same direction`	(L1 + L2)/(s1 - s2).
`After meeting, take a and b hours to finish`	s1 : s2 = √(b) : √(a).

Shortcuts

Pole and platform together Use when: 'Crosses a pole in 15 s and a 100 m platform in 25 s'.

1. The extra 10 s is the platform: speed = 100/10 = 10 m/s.
2. Length = 10 × 15 = 150 m.

Try it: As above. Train length?

1. 150 m.

Answer: 150 m

Check yourself

- You can add the platform length when crossing it.
- You can add both lengths when two trains pass.
- You can use relative speed for trains and walkers.

Class plan (2 hours)

`10 min`	Warm-up: km/h to m/s.
`30 min`	Pole vs platform with the diagrams.
`30 min`	Two trains, both directions; a man walking.
`40 min`	Practice set.
`10 min`	Recap.

Practice: 9 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/problems-on-trains

Boats and Streams

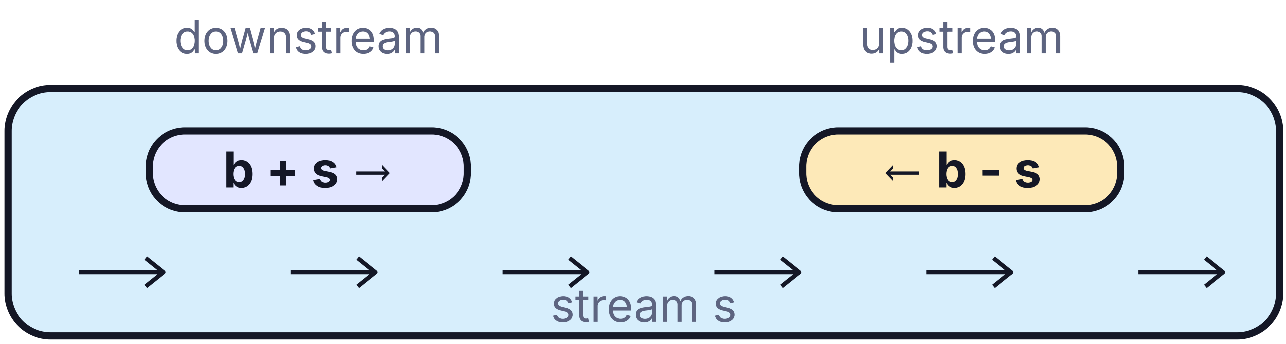
> **Rowing with the river's flow, the water carries you: you go faster. Against it, the water pushes back: you go slower.**

Boats and streams is relative speed with the river as the second mover. Two formulas cover every question.

Downstream and upstream

Boat speed in still water b, stream speed s. Downstream (with the flow) = b + s. Upstream (against) = b - s.

Going back: b = (down + up) / 2, s = (down - up) / 2.



Downstream = boat + stream. Upstream = boat - stream

`Down = b + s, Up = b - s`	
`b = (D + U)/2, s = (D - U)/2`	Half the sum, half the difference.

Example 1: A boat goes 24 km downstream in 2 hours and 12 km upstream in 2 hours. Find the boat's speed and the stream's speed.

1. Down = 12 km/h, Up = 6 km/h.
   Why: Distance / time for each.
2. b = (12 + 6)/2 = 9. s = (12 - 6)/2 = 3.
3. Boat 9 km/h, stream 3 km/h.

Answer: **Boat 9 km/h, stream 3 km/h**

Example 2: A man rows at 5 km/h in still water; the river flows at 1 km/h. It takes him 75 minutes to row to a place and back. How far is the place?

1. Down 6 km/h, up 4 km/h.
2. d/6 + d/4 = 5/4 hours.
   Why: 75 minutes = 5/4 h.
3. (2d + 3d)/12 = 5/4, so 5d = 15.
   Why: Common denominator 12.
4. d = 3 km.

Answer: 3

Formula summary

`Downstream`	b + s.
`Upstream`	b - s.
`Boat in still water`	(down + up)/2.
`Stream`	(down - up)/2.
`Round trip time`	d/(b + s) + d/(b - s).

Shortcuts

Speeds from distances Use when: '20 km down in 2 h, 12 km up in 2 h'.

1. Down 10, up 6.
2. Boat 8, stream 2.

Try it: Boat speed?

1. (10 + 6)/2.

Answer: 8 km/h

Check yourself

- You can write downstream and upstream speeds.
- You can get boat and stream speeds from them.
- You can solve round-trip time questions.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Down and up with the river picture.
`25 min`	Finding b and s.
`20 min`	Round trips.

`30 min`	Practice set.
`5 min`	Recap.

Practice: 4 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/boats-and-streams

Averages

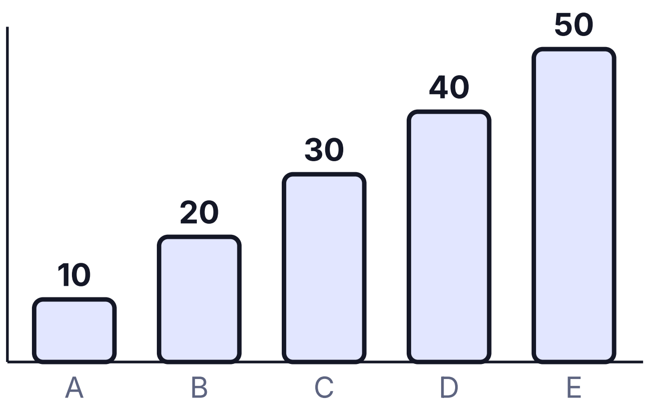
> **Five friends have Rs 10, 20, 30, 40 and 50. If they pool it and share equally, each gets Rs 30. The average is the 'equal share'.**

Averages appear on their own and inside DI, mixtures and ages. Most questions are one identity: total = average × count.

Average = total / count, so total = average × count

The average is what everyone would have if the total were shared equally.

The key move in almost every question is to go to totals: total = average × count. Totals can be added and subtracted; averages cannot.



Average 30: the bars above 30 give exactly what the bars below need

Example 1: The average of 6 numbers is 24. If one number is removed, the average of the rest is 22. What number was removed?

1. Total of 6 = 6 × 24 = 144.
   Why: Go to totals.
2. Total of 5 = 5 × 22 = 110.
3. Removed = 144 - 110 = 34.

Answer: 34

Someone joins or leaves

When a person joins and the average changes, compare totals. Or think of the newcomer as 'giving' or 'taking' from everyone.

A new person raises the average of 10 people by 2: the newcomer brings the old average plus 2 for each of the 11 people now in the group.

Example 1: The average age of 30 students is 15. When the teacher's age is included, the average becomes 16. Teacher's age?

1. Old total = 30 × 15 = 450.
2. New total = 31 × 16 = 496.
   Why: 31 people now.
3. Teacher = 496 - 450 = 46.

Answer: 46

Combined averages and consecutive numbers

Two groups together: combined average = (n1 a1 + n2 a2) / (n1 + n2). It lies between the two averages, closer to the bigger group.

Consecutive numbers (or any evenly spaced list): the average is the middle term, which is also (first + last)/2.

Example 1: Class A has 30 students averaging 60 marks; class B has 20 averaging 70. Combined average?

1. Totals: 1800 and 1400.
   Why: Average × count.
2. 3200 / 50 = 64.

Answer: 64

Example 2: The average of 7 consecutive odd numbers is 27. Find the largest.

1. The middle (4th) number is 27.
   Why: Evenly spaced: the average is the middle.
2. Largest = 27 + 3 × 2 = 33.

Answer: 33

Formula summary

`Average = sum / count`	
`Assumed mean (deviation) method`	Pick a round number, add the + and - gaps, divide by count, add back.
`Someone joins, average rises by d`	Newcomer = old average + (new count) × d.
`Someone leaves, average rises by d`	Leaver = old average - (new count) × d.
`Consecutive numbers`	Average = middle term = (first + last)/2.
`Overlapping groups (first 6, last 6 of 11)`	Middle term = sum of both groups - total.

Shortcuts

Deviation method Use when: Five 2- or 3-digit numbers clustered together.

1. Assume 100: 92, 97, 105, 108, 88 → -8, -3, +5, +8, -12 = -10.
2. -10/5 = -2 → 98.

Try it: Average of 92, 97, 105, 108, 88

1. 98.

Answer: 98

New member shifts the average Use when: Teacher joins a class, batsman plays one more innings.

1. Everyone, old and new, gets +d. The newcomer paid for all of it.

Try it: 30 students average 45. With the teacher the average is 46. Teacher's age?

1. 45 + 31 × 1.

Answer: 76

Common mistakes

Wrong: Averaging two averages when the groups have different sizes.	**Instead:** Weight by group size.

Check yourself

- You can always move to totals.
- You can find the age or mark of someone who joins or leaves.
- You can combine two groups' averages.

Class plan (2 hours)

`10 min`	Warm-up.
`20 min`	Average as equal share; totals.
`25 min`	Removing, replacing, joining.
`20 min`	Combined groups; consecutive numbers.
`10 min`	Replacement: a wrong mark corrected.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 11 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/averages

Ages

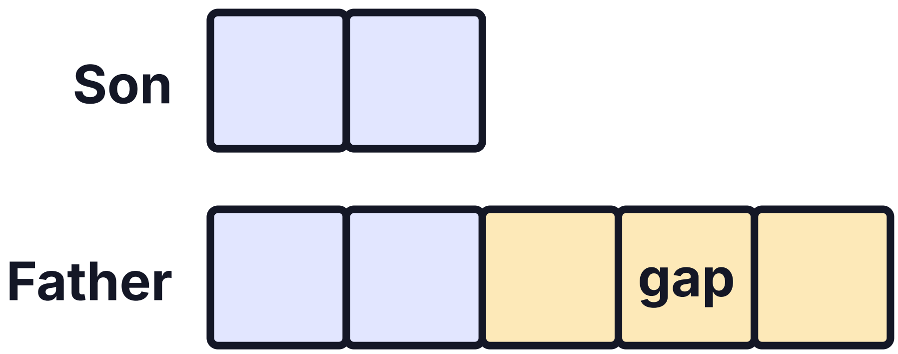
> **Everyone gets older at the same speed. So the DIFFERENCE between two people's ages never changes.**

Age questions are ratio and linear equations dressed up. Use the fact that the age gap is fixed, or just try the options.

The fixed gap, and the bar model

If a father is 30 years older than his son now, he was 30 years older 10 years ago and will be 30 years older in 10 years.

Ratios change with time; the gap does not. Put the gap in the bar model and one box's value is found.



Now 2 : 5. The 3-box gap is fixed forever

Example 1: The ratio of a father's age to his son's is 5 : 2. The father is 30 years older. Their ages?

1. The gap is 5 - 2 = 3 parts.
   Why: Bar model: 3 boxes.
2. 3 parts = 30, so 1 part = 10.
3. Father 50, son 20.

Answer: 50

Example 2: Four years ago, A was 3 times as old as B. In 4 years A will be twice as old as B. A's present age?

1. Let B four years ago = x, A = 3x. Now: A = 3x + 4, B = x + 4.
   Why: Start from the time the question describes first.
2. In 4 years: 3x + 8 = 2(x + 8).
3. x = 8.
4. A now = 28.

Answer: 28

Try the options

Age questions always have whole-number answers, and the options test directly: put the option in, check every sentence.

Example 1: The sum of the ages of a mother and daughter is 50. Five years ago the mother was 7 times as old as the daughter. Mother's age? (a) 35 (b) 40 (c) 42 (d) 45

1. Try 40: daughter 10. Five years ago: 35 and 5. 35 = 7 × 5. Fits.
   Why: Check both sentences.
2. 40.

Answer: 40

Formula summary

`Age difference is constant`	If ratio changes from a:b to c:d, the difference in units must match: scale both ratios to the same difference.
`n years ago / hence`	Subtract / add n to EVERY person.
`Sum after n years`	Sum + n × (number of people).
`Children born at intervals`	Ages x, x + k, x + 2k ... sum = n × + k(0 + 1 + ... + n - 1).

Shortcuts

Ratio-units trick (no equations) Use when: 'Ratio now 4:5, 5 years ago 7:9'.

1. Differences: 5 - 4 = 1 unit, 9 - 7 = 2 units. Make them equal: now 8:10.
2. Now 8:10, then 7:9. Each person dropped 1 unit = 5 years.
3. So 1 unit = 5, ages 40 and 50.

Try it: A:B = 4:5 now, 7:9 five years ago. B's age?

1. Scale to 8:10 vs 7:9.
2. 1 unit = 5 years.
3. B = 10 × 5.

Answer: 50

Times-older problems Use when: 'Father is 3 times son now; in 15 years, twice'.

1. Now: 3:1, gap 2 units. Later 2:1, gap 1 unit → scale to 4:2.
2. Now 3:1, later 4:2: son went 1 → 2 units = 15 years.
3. Son 15, father 45.

Try it: Father's age?

1. 45.

Answer: 45

Check yourself

- You can use the fixed age gap.
- You can turn ratio statements into parts.
- You can check options against every sentence.

Class plan (2 hours)

`10 min`	Warm-up.
`25 min`	Fixed gap; bar model.
`30 min`	Past and future equations.
`20 min`	Try the options.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 11 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/ages

Mixtures and Alligation

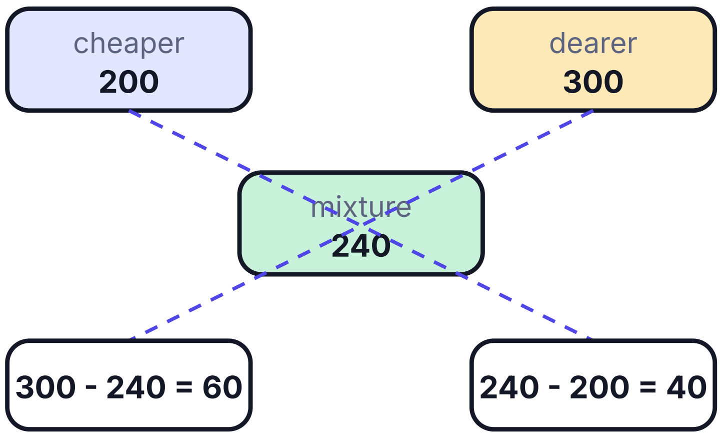
> **Mix tea at Rs 200 a kg with tea at Rs 300 a kg and sell the mix at Rs 240. How much of each did you use? Alligation answers that with one cross.**

Mixture questions (milk and water, two kinds of rice, alloys, even a class's average) all reduce to the alligation cross or the removal formula.

The alligation cross

Write the cheaper and dearer prices at the top, the mixture's price in the middle. Cross-subtract: cheaper : dearer = (dearer - mean) : (mean - cheaper).

It works for any weighted average: prices, percentages of milk, speeds over equal times, marks of two classes.



Cross-subtract: cheaper : dearer = 60 : 40

Example 1: In what ratio must tea at Rs 200/kg be mixed with tea at Rs 300/kg so the mixture is worth Rs 240/kg?

1. Cross: (300 - 240) : (240 - 200) = 60 : 40.
   Why: Distance of the mean from the OTHER price.
2. 3 : 2.

Answer: 3 : 2

Removal and replacement

A vessel has x litres of milk. y litres are taken out and replaced with water, n times. Milk left = x (1 - y/x)^n.

Each time, the same fraction of whatever milk is left goes out.

$\text{Milk left} = x\,(1 - y/x)^n$	x = vessel size, y = amount removed each time, n = number of times.

Example 1: A 40-litre can is full of milk. 4 litres are removed and replaced with water; this is done 3 times. How much milk is left?

1. Each time 4/40 = 1/10 of the liquid goes, so 9/10 of the milk stays.
2. After 3 times: $40 \times (9/10)^3 = 40 \times 0.729$.
3. = 29.16 litres.

Answer: 29.16

Mixing two mixtures

Add up each ingredient separately. Mixture 1 has milk : water = 3 : 1 and mixture 2 has 5 : 3; take equal amounts, say 8 litres each: milk 6 + 5 = 11, water 2 + 3 = 5. New ratio 11 : 5.

Example 1: Two vessels have milk and water in ratios 3 : 1 and 5 : 3. Equal quantities are mixed. New ratio?

1. Take 8 litres of each (LCM of 4 and 8).
   Why: Pick a size that splits both ratios evenly.
2. Milk: 6 + 5 = 11. Water: 2 + 3 = 5.
3. 11 : 5.

Answer: 11 : 5

Formula summary

`Alligation`	Cheap qty : Dear qty = (Dear - Mean) : (Mean - Cheap).
`Repeated removal and replacement`	Pure left = Initial × $(1 - x/V)^n$.
`Water added, sold at CP, gain g%`	Water : Milk = g : 100.
`Adding one component to shift ratio`	Hold the other component fixed and rescale.

Shortcuts

Cross (see-saw) diagram Use when: Two prices, one mean.

1. Write cheap and dear on top, mean in the middle.
2. Cross-subtract to get the ratio.

Try it: Rice at 60 and 75 mixed to sell at 65. Ratio?

1. (75 - 65):(65 - 60) = 10:5.

Answer: 2:1

Check yourself

- You can use the cross for any two-ingredient average.
- You can apply $x(1 - y/x)^n$ for repeated removal.

- You can combine two mixtures ingredient by ingredient.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Alligation cross: prices, percentages, averages.
`25 min`	Removal and replacement.
`20 min`	Mixing mixtures.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 10 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/mixtures-and-alligation

Mensuration

> **How much paint for a wall, how much water in a tank, how much wire around a field? Mensuration is measuring. Every question asks one of three things: the length around (perimeter), the flat space inside (area), or the space a solid fills (volume).**

Placement papers ask 1 or 2 mensuration questions: areas of triangles, rectangles, circles and trapeziums, volumes of cylinders and cones, and 'melt one solid into another'. Once you can picture the shape, each question is one formula. So this lesson draws every shape first, and says what each letter in the formula stands for.

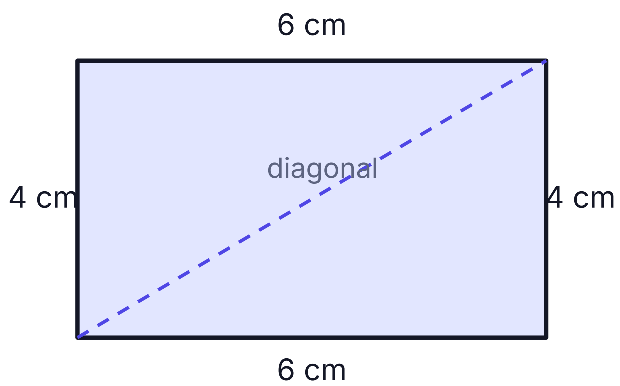
Perimeter, area, volume: what they measure

Perimeter is the distance around a flat shape. Walk around a field and count your steps. It is a length, measured in cm or m.

Area is how much flat surface a shape covers. Count how many 1 cm × 1 cm squares fit inside. It is measured in square units: cm^2, m^2.

Volume is how much space a solid takes up. Count how many 1 cm cubes fit inside. It is measured in cubic units: cm^3, m^3. Capacity (how much liquid it holds) is volume too: 1000 cm^3 = 1 litre.

Surface area is the total area of all the faces of a solid: the paint you would need to cover a box. 'Curved surface area' (CSA) counts only the curved part; 'total surface area' (TSA) adds the flat ends as well.



A 6 cm × 4 cm rectangle: perimeter 20 cm (walk around), area 24 cm^2 (24 unit squares fit inside)

Measure	Units	Think of it as
Perimeter	cm, m	fence around a field
Area	cm^2, m^2	tiles on a floor
Volume	cm^3, m^3, litres	water in a tank
Surface area	cm^2, m^2	paint on a box

Example 1: A rectangular field is 50 m long and 30 m wide. How much fencing goes around it, and how much grass is inside?

1. Perimeter = 2 × (50 + 30) = 160 m.
   Why: Two lengths and two breadths.
2. Area = 50 × 30 = 1500 m^2.
   Why: Length times breadth counts the unit squares.
3. Fencing 160 m, grass 1500 m^2.

Answer: **Fencing 160 m, grass 1500 m^2**

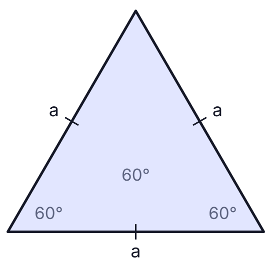
Exam tip: Check the units in the options first. If the question asks for area and an option says 'm', that option is wrong.

Triangles: the six types

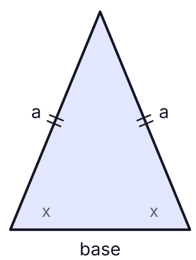
Triangles are named in two ways. By their sides: equilateral (all three sides equal), isosceles (two sides equal), scalene (all different). By their biggest angle: acute (every angle under 90 degrees), right-angled (one angle exactly 90), obtuse (one angle over 90).

The three angles of every triangle add up to 180 degrees. So an equilateral triangle has three 60-degree angles, and an isosceles triangle has its two base angles equal.

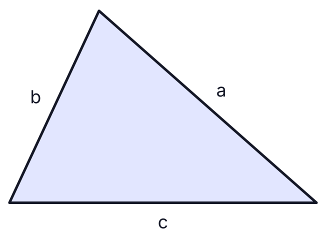
A triangle can be both kinds at once: a right-angled isosceles triangle has angles 90, 45, 45.



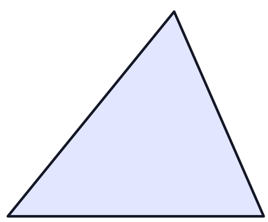
Equilateral: all 3 sides equal, every angle 60°



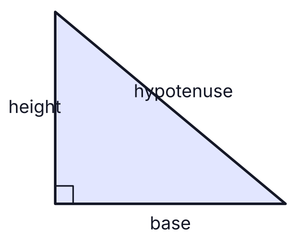
Isosceles: 2 sides equal, the 2 base angles equal



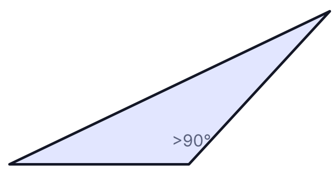
Scalene: all 3 sides different



Acute-angled: every angle below 90°



Right-angled: one angle is 90°



Obtuse-angled: one angle above 90°

`Angle sum = 180°`	Any triangle.
`Any two sides together are longer than the third`	3, 4, 8 cannot make a triangle because 3 + 4 < 8.

`Biggest side faces the biggest angle`	In a right triangle the side opposite 90° (hypotenuse) is the longest.

Example 1: Two angles of a triangle are 48° and 67°. Find the third angle and say what type the triangle is by its angles.

1. Third angle = 180 - 48 - 67 = 65°.
   Why: The three angles always add to 180.
2. All three angles (48, 67, 65) are below 90.
   Why: The type by angle depends on the biggest angle.
3. 65°, and the triangle is acute-angled.

Answer: 65

Exam tip: In a 'which type' question, look only at the biggest angle: under 90 acute, exactly 90 right, over 90 obtuse.

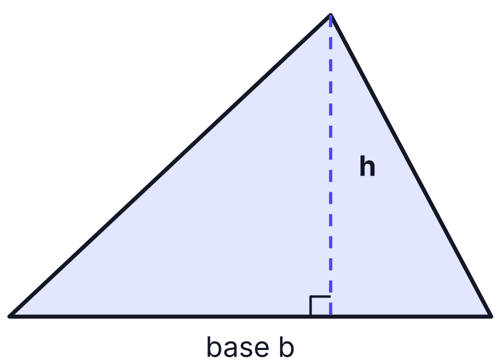
Area of a triangle, and right triangles

Area of any triangle = 1/2 × base × height. The height is the perpendicular (straight-down) distance from the top corner to the base, not a slanting side. A triangle is exactly half of a rectangle with the same base and height; that is where the 1/2 comes from.

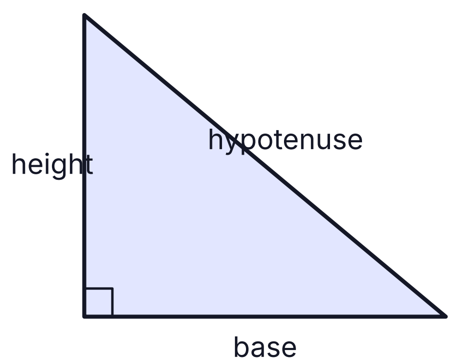
If you only know the three sides, use Heron's formula: find s = half the perimeter, then area = √(s(s - a)(s - b)(s - c)).

In a right-angled triangle, Pythagoras connects the sides: hypotenuse^2 = base^2 + height^2. Some whole-number triples come up again and again. Learn them and many questions need no square roots at all.

Equilateral triangle with side a: height = (√(3)/2) × a, area = (√(3)/4) × a^2.



Area = 1/2 × base × height. The height is the perpendicular, not a side



Right triangle: hypotenuse^2 = base^2 + height^2

Pythagorean triple (a, b, hypotenuse)	Multiples also work
3, 4, 5	6, 8, 10 · 9, 12, 15
5, 12, 13	10, 24, 26 · 15, 36, 39

8, 15, 17	16, 30, 34 · 24, 45, 51
7, 24, 25	14, 48, 50 · 21, 72, 75
20, 21, 29	40, 42, 58 · 60, 63, 87
12, 35, 37	24, 70, 74 · 36, 105, 111
9, 40, 41	18, 80, 82 · 27, 120, 123

`Area = 1/2 × base × height`	Height must be perpendicular to the base.
`Heron: s = (a + b + c)/2, area = √(s(s-a)(s-b)(s-c))`	When only the three sides are given.
Equilateral: area = $(\sqrt{3}/4)\, a^2$, height = $(\sqrt{3}/2)\, a$	One side is enough.
`Right triangle: area = 1/2 × (the two shorter sides)`	The two legs are base and height of each other.
Isosceles with equal sides a and base b: area = $(b/4)\sqrt{4a^2 - b^2}$	Drop the height to the middle of the base.

Example 1: Find the area of a triangle with sides 13 cm, 14 cm and 15 cm.

1. s = (13 + 14 + 15)/2 = 21.
   Why: Heron needs half the perimeter.
2. s - a = 8, s - b = 7, s - c = 6.
   Why: Subtract each side from s.
3. Area = √(21 × 8 × 7 × 6) = √(7056) = 84 cm^2.
   Why: Pair factors: 21 × 8 × 7 × 6 = (7 × 3)(2 × 4)(7)(2 × 3) = $7^2 \times 2^4 \times 3^2$, whose root is 7 × 4 × 3 = 84.

Answer: 84

Example 2: The hypotenuse of a right triangle is 25 cm and one side is 7 cm. Find its area.

1. Other side = $\sqrt{25^2 - 7^2}$ = √(576) = 24.
   Why: Or spot the triple 7, 24, 25.
2. Area = 1/2 × 7 × 24 = 84 cm^2.
   Why: The two shorter sides are base and height.

Answer: 84

Example 3: An equilateral triangle has side 8 cm. Find its area.

1. Area = $(\sqrt{3}/4) \times 8^2 = (\sqrt{3}/4) \times 64$.
   Why: Formula for equilateral triangles.
2. = $16\sqrt{3}$ cm^2 (about 27.7 cm^2).

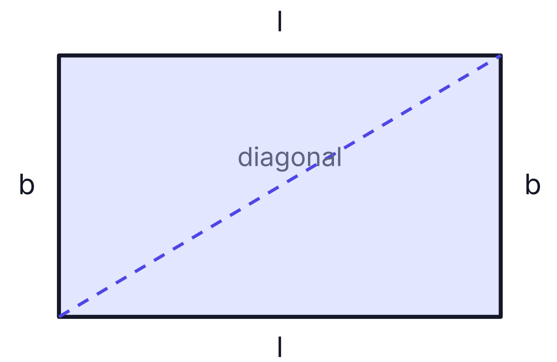
Answer: $16\sqrt{3}$

Exam tip: See 5, 12, 13 or 8, 15, 17 hiding in the numbers? It is a right triangle; skip the square root.

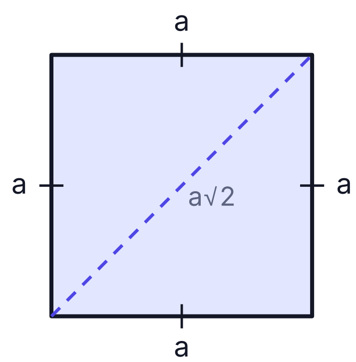
Square and rectangle

A rectangle has opposite sides equal and all angles 90 degrees. Area = length × breadth. Perimeter = 2(length + breadth). The diagonal cuts it into two right triangles, so diagonal = $\sqrt{l^2 + b^2}$.

A square is a rectangle with all four sides equal. Area = a^2. Perimeter = 4a. Diagonal = a × √(2). If you know only the diagonal d, the area is $d^2 / 2$.



Rectangle: area = l × b, perimeter = 2(l + b), diagonal = $\sqrt{l^2 + b^2}$



Square: area = a^2 = (diagonal^2)/2, perimeter = 4a, diagonal = a × √(2)

Rectangle: area lb, perimeter 2(l + b), diagonal $\sqrt{l^2 + b^2}$	
Square: area $a^2 = d^2/2$, perimeter 4a, diagonal a √(2)	
Path of width w around a rectangle (outside)	Area of path = 2w(l + b + 2w)

Example 1: The diagonal of a square is 12 cm. Find its area.

1. Area = d^2 / 2 = 144 / 2 = 72 cm^2.
   Why: A square is two right isosceles triangles on the diagonal; this formula skips finding the side.

Answer: 72

Example 2: A garden 40 m × 30 m has a path 2 m wide all around it on the outside. Find the area of the path.

1. Outer rectangle: (40 + 4) × (30 + 4) = 44 × 34 = 1496 m^2.
   Why: The path adds 2 m on each side, so 4 m to each dimension.
2. Garden: 40 × 30 = 1200 m^2.
   Why: The inner part.
3. Path = 1496 - 1200 = 296 m^2.

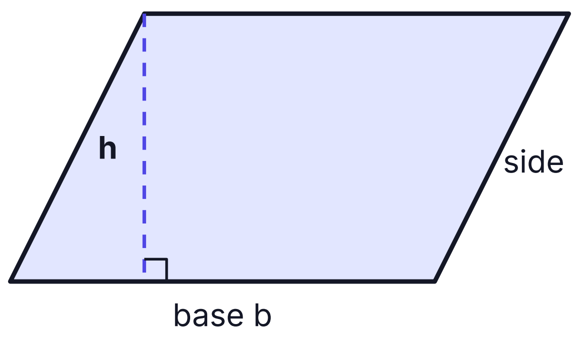
Answer: 296

Parallelogram

A parallelogram is a 'pushed-over' rectangle: opposite sides are parallel and equal, but the corners are not 90 degrees.

Area = base × height, where the height is the perpendicular distance between the two parallel sides. Cut the triangle off one end and slide it to the other and you get a rectangle with the same base and height. That is why.

The slanted side is NOT the height. Using it is the most common mistake here.



Parallelogram: area = base × height (the height, NOT the slanted side)

`Area = base × height`	Perpendicular height, not the slanted side.
`Perimeter = 2(a + b)`	Two pairs of equal sides.
`Diagonals bisect each other`	Each diagonal cuts the other in half.

Example 1: A parallelogram has base 15 cm and the perpendicular height to it is 8 cm. Its slanted side is 10 cm. Find its area and perimeter.

1. Area = 15 × 8 = 120 cm^2.
   Why: Use the perpendicular height; ignore the 10 cm side for area.
2. Perimeter = 2(15 + 10) = 50 cm.
   Why: Perimeter uses the actual sides.

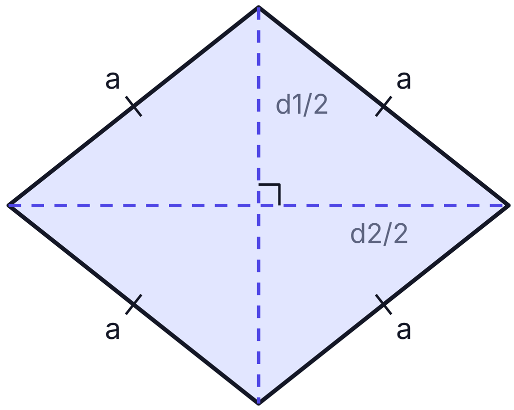
Answer: **Area 120 cm^2, perimeter 50 cm**

Rhombus

A rhombus is a parallelogram with all four sides equal: a diamond. Its two diagonals cut each other at right angles and in half.

Area = 1/2 × d1 × d2 (half the product of the diagonals). The diagonals split the rhombus into 4 right triangles, and together those 4 triangles make half of the rectangle d1 × d2.

Because the diagonals meet at 90 degrees, $side^2 = (d1/2)^2 + (d2/2)^2$. That lets you get the side from the diagonals.



Rhombus: 4 equal sides, diagonals cut at 90°. Area = 1/2 × d1 × d2

`Area = 1/2 × d1 × d2`	Also = side × height, like any parallelogram.
$\text{Side} = \sqrt{(d1/2)^2 + (d2/2)^2}$	Pythagoras on one of the four small right triangles.
`Perimeter = 4 × side`	

Example 1: The diagonals of a rhombus are 16 cm and 12 cm. Find its area, side and perimeter.

1. Area = 1/2 × 16 × 12 = 96 cm^2.
   Why: Half the product of the diagonals.
2. Half-diagonals are 8 and 6, so side = √(64 + 36) = 10 cm.
   Why: The diagonals meet at 90°: a 6-8-10 right triangle.
3. Perimeter = 4 × 10 = 40 cm.
   Why: All four sides equal.

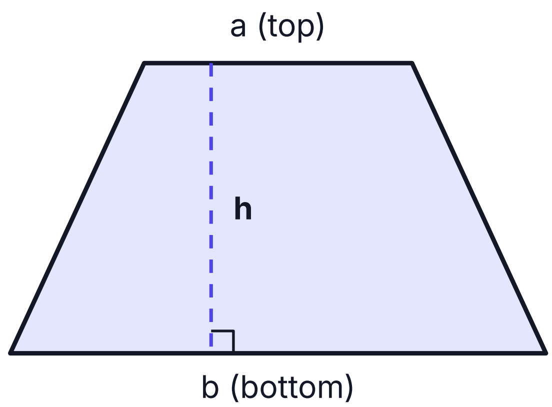
Answer: **Area 96 cm^2, side 10 cm, perimeter 40 cm**

Trapezium

A trapezium has exactly one pair of parallel sides (the top a and bottom b), with a perpendicular height h between them.

Area = 1/2 × (a + b) × h: the average of the two parallel sides times the height. Put two copies of the trapezium together, one upside down, and they make a parallelogram with base (a + b) and height h. Half of that is one trapezium.

If the non-parallel sides are equal, it is an isosceles trapezium; drop two heights and the bottom splits into a, and two equal pieces of (b - a)/2.



Trapezium: one pair of parallel sides. Area = 1/2 × (a + b) × h

`Area = 1/2 × (sum of parallel sides) × height`	(a + b)/2 is the 'average width'.
`Isosceles trapezium: each overhang = (b - a)/2`	Then height = $\sqrt{(\text{slant}^2 - \text{overhang}^2)}$.

Example 1: The parallel sides of a trapezium are 12 cm and 20 cm, and the distance between them is 7 cm. Find its area.

1. Area = 1/2 × (12 + 20) × 7.
   Why: Add the parallel sides, halve, times height.
2. = 16 × 7 = 112 cm^2.

Answer: 112

Example 2: An isosceles trapezium has parallel sides 10 cm and 22 cm, and each slanted side is 10 cm. Find its area.

1. Overhang on each side = (22 - 10)/2 = 6 cm.
   Why: The longer base sticks out equally on both sides.
2. Height = $\sqrt{(10^2 - 6^2)}$ = 8 cm.
   Why: Each slanted side is the hypotenuse of a 6-8-10 triangle.
3. Area = 1/2 × (10 + 22) × 8 = 128 cm^2.

Answer: 128

Exam tip: Trapezium area = average of the parallel sides × height. Say it that way and you will not forget the 1/2.

Circle, semicircle, sector and ring

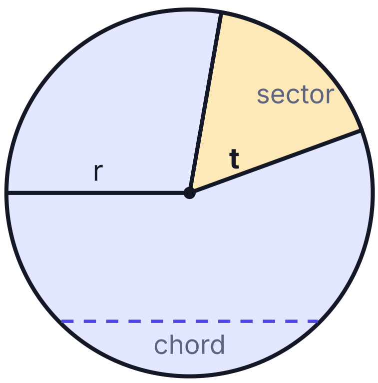
Every point on a circle is the same distance r (radius) from the centre. The diameter d = 2r goes all the way across.

Circumference (perimeter) = 2 π r. Area = π r^2. Use π = 22/7 when the radius is a multiple of 7, otherwise 3.14.

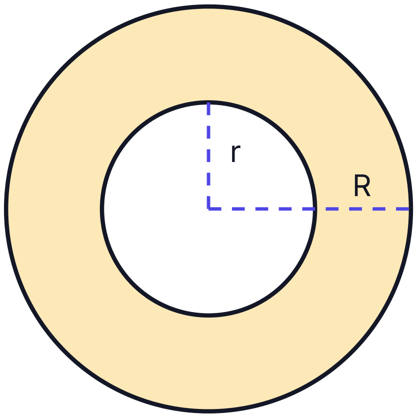
A sector is a slice, like a pizza slice, with angle t at the centre. It is t/360 of the whole circle, so its area is (t/360) × πr^2 and its curved edge (arc) is (t/360) × 2 π r.

A semicircle is a sector of 180 degrees. Its perimeter is the curved half (π r) PLUS the diameter (2r). Forgetting the diameter is a classic trap.

A ring (a circular path) is a big circle minus a small one: $\pi (R^2 - r^2)$.



Circle: area = πr^2, circumference = 2 π r. Sector with angle t: (t/360) × πr^2



Ring (annulus): area = $\pi (R^2 - r^2) = \pi (R + r)(R - r)$

Circumference = 2 π r = π d	
Area = πr^2	
Sector area = (t/360) πr^2, arc = (t/360) 2 π r	t = angle at the centre.
Semicircle perimeter = π r + 2r	Curved part plus the straight diameter.
Ring area = $\pi (R^2 - r^2) = \pi (R + r)(R - r)$	The factorised form avoids big squares.
A wheel of radius r covers 2 π r in one turn	Distance = turns × circumference.

Example 1: The circumference of a circle is 44 cm. Find its area (π = 22/7).

1. 2 × 22/7 × r = 44, so r = 7 cm.
   Why: Get the radius first; everything about a circle comes from r.
2. Area = 22/7 × 7 × 7 = 154 cm^2.

Answer: 154

Example 2: Find the area of a sector of angle 90° in a circle of radius 14 cm (π = 22/7).

1. A 90° sector is 90/360 = 1/4 of the circle.
   Why: Sectors are fractions of the full circle.
2. Area = 1/4 × 22/7 × 14 × 14 = 154 cm^2.

Answer: 154

Example 3: A circular park of radius 20 m has a path 7 m wide around it on the outside. Find the area of the path ($\pi = 22/7$).

1. Outer radius R = 27, inner r = 20.
   Why: The path adds 7 m to the radius.
2. Area = $22/7 \times (27 + 20)(27 - 20) = 22/7 \times 47 \times 7$.
   Why: Ring formula in factorised form; the 7 cancels.
3. = $22 \times 47 = 1034$ m^2.

Answer: 1034

Example 4: A wheel of radius 35 cm makes how many turns to cover 1.1 km ($\pi = 22/7$)?

1. One turn = $2 \times 22/7 \times 35 = 220$ cm.
   Why: Circumference is the distance per turn.
2. 1.1 km = 110000 cm.
   Why: Same units before dividing.
3. Turns = 110000 / 220 = 500.

Answer: 500

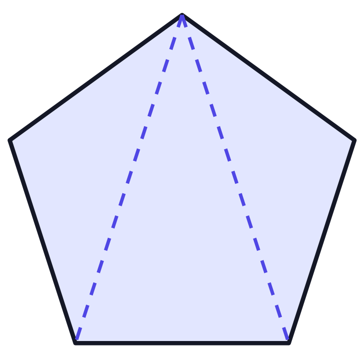
Polygons

A polygon is any closed shape with straight sides: pentagon (5), hexagon (6), octagon (8). 'Regular' means all sides and all angles are equal.

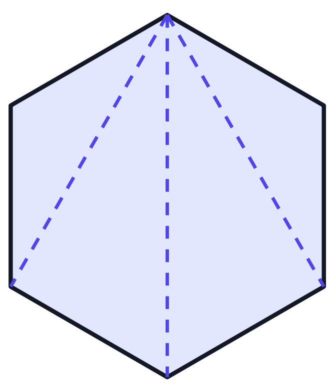
From one corner you can cut an n-sided polygon into $(n - 2)$ triangles. Each triangle has 180 degrees, so the interior angles add up to $(n - 2) \times 180$. In a regular polygon each interior angle is that divided by n.

The exterior angles (the turn you make at each corner walking around) always add to 360, so each exterior angle of a regular polygon is 360/n.

A regular hexagon is 6 equilateral triangles, so its area is $6 \times (\sqrt{3}/4)\, a^2 = (3\sqrt{3}/2)\, a^2$.



5 sides: cut into 3 triangles from one corner, so angles add to 540°



6 sides: cut into 4 triangles from one corner, so angles add to 720°

Sides n	Angle sum	Each interior angle (regular)	Each exterior angle	Diagonals
3	180	60	120	0
4	360	90	90	2
5	540	108	72	5
6	720	120	60	9
8	1080	135	45	20
10	1440	144	36	35
12	1800	150	30	54

Sum of interior angles = (n - 2) × 180	
Each exterior angle (regular) = 360/n	Interior + exterior = 180.
Diagonals = n(n - 3)/2	Each corner joins n - 3 others; each diagonal counted twice.
Regular hexagon area = (3 √(3)/2) a^2	Six equilateral triangles.

Example 1: Each interior angle of a regular polygon is 140°. How many sides does it have?

1. Each exterior angle = 180 - 140 = 40°.
   Why: Interior and exterior angles at a corner make a straight line.
2. n = 360 / 40 = 9.
   Why: Exterior angles add to 360.

Answer: 9

Exam tip: For 'how many sides' questions go through the exterior angle: n = 360 / (180 - interior).

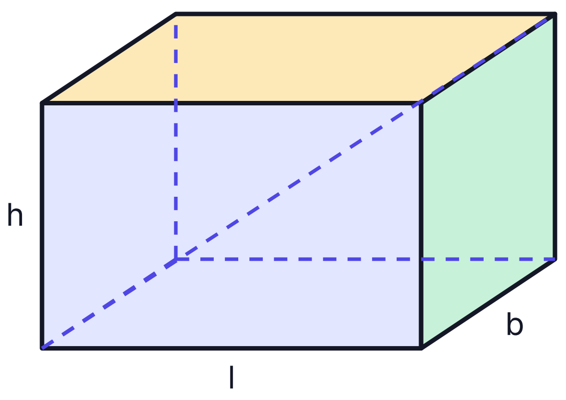
Cuboid and cube

A cuboid is a box with length l, breadth b and height h. Volume = l × b × h: the number of 1 cm cubes that fill it.

It has 6 rectangular faces in 3 matching pairs (top/bottom, front/back, left/right). Total surface area = 2(lb + bh + hl). The four walls of a room without floor and ceiling (lateral surface) = 2h(l + b).

The longest rod that fits inside goes corner to corner: diagonal = $\sqrt{l^2 + b^2 + h^2}$.

A cube has l = b = h = a: volume a^3, surface $6a^2$, diagonal $a\sqrt{3}$.



Cuboid: volume = l × b × h, total surface = 2(lb + bh + hl), diagonal = √($l^2 + b^2 + h^2$)

`Cuboid: volume lbh, TSA 2(lb + bh + hl), walls 2h(l + b), diagonal √(l² + b² + h²)`	
`Cube: volume a³, TSA 6a², diagonal a √(3)`	
`Number of small cubes = big volume / small volume`	Only if the sides divide exactly; otherwise count along each edge.

Example 1: A room is 10 m long, 8 m wide and 4 m high. Find the cost of painting its four walls at Rs 20 per m^2.

1. Area of four walls = 2 × 4 × (10 + 8) = 144 m^2.
   Why: Walls only: no floor, no ceiling.
2. Cost = 144 × 20 = Rs 2880.

Answer: 2880

Example 2: How many 2 cm cubes can be cut from a cuboid 10 cm × 8 cm × 6 cm?

1. Along each edge: 10/2 = 5, 8/2 = 4, 6/2 = 3.
   Why: Count how many fit on each edge.
2. 5 × 4 × 3 = 60 cubes.

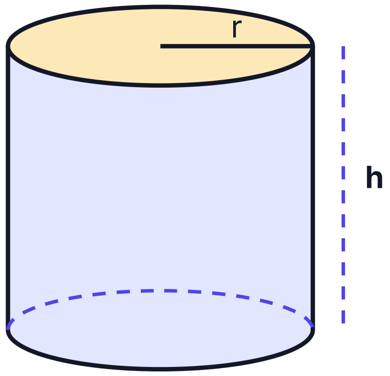
Answer: 60

Cylinder

A cylinder is a circle pulled straight up to height h: a pipe, a tin, a drum. Volume = base area × height = $\pi r^2 h$.

Unroll the curved side and it becomes a rectangle: one side is the circumference $2 \pi r$, the other is h. So curved surface area = $2 \pi r h$. Add the two circular ends for the total: $2 \pi r h + 2 \pi r^2 = 2 \pi r (r + h)$.

A hollow pipe with outer radius R and inner radius r has volume of material $\pi (R^2 - r^2) h$.



Cylinder: volume = $\pi r^2 h$, curved surface = $2 \pi r h$, total = $2 \pi r (r + h)$

Volume = $\pi r^2 h$	Area of the circle × height.
CSA = $2 \pi r h$	The label on a tin.
TSA = $2 \pi r (r + h)$	Label plus the two lids.
Hollow: $\pi (R^2 - r^2) h$	Metal in a pipe.

Example 1: A cylindrical tank has radius 7 m and height 10 m. How many litres does it hold (π = 22/7)?

1. Volume = 22/7 × 7 × 7 × 10 = 1540 m^3.
   Why: $\pi r^2 h$.
2. 1 m^3 = 1000 litres.
   Why: Convert at the end.
3. 1540 × 1000 = 15,40,000 litres.

Answer: 1540000

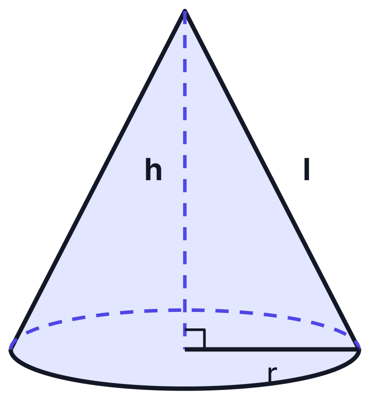
Cone

A cone is a circle that narrows to a point: an ice-cream cone, a tent. Its height h goes straight up the middle; its slant height l runs down the side.

h, r and l make a right triangle, so $l = \sqrt{(r^2 + h^2)}$.

A cone holds exactly one third of a cylinder with the same base and height: volume = $(1/3) \pi r^2 h$. (Fill a cone with water and pour it into the matching cylinder: it takes three cones to fill it.)

Curved surface area = $\pi r l$. Total = $\pi r (l + r)$.



Cone: volume = 1/3 π r^2 h, slant l = √(r^2 + h^2), curved surface = π r l

`Volume = (1/3) π r² h`	One third of the matching cylinder.
`Slant l = √(r² + h²)`	
`CSA = π r l, TSA = π r (l + r)`	Surface uses the slant height, volume uses the vertical height.

Example 1: A cone has radius 6 cm and height 8 cm. Find its curved surface area, in terms of π.

1. Slant l = √(36 + 64) = 10 cm.
   Why: A 6-8-10 right triangle.
2. CSA = π × 6 × 10 = 60 π cm^2.

Answer: 60 π

Example 2: A conical tent has radius 7 m and height 24 m. Find the canvas needed (π = 22/7).

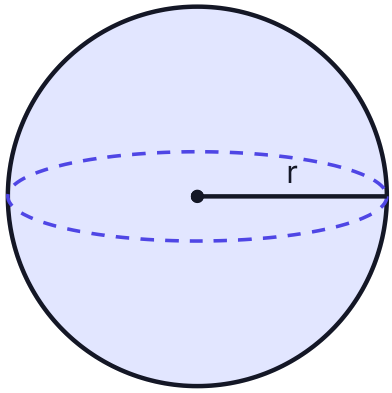
1. Slant = √(49 + 576) = 25 m.
   Why: The 7-24-25 triple.
2. Canvas = CSA = 22/7 × 7 × 25 = 550 m^2.
   Why: A tent has no floor, so only the curved surface.

Answer: 550

Sphere and hemisphere

A sphere is a perfect ball: every point on it is r from the centre. Volume = (4/3) π r^3. Surface area = 4 π r^2 (exactly four circles of the same radius).

A hemisphere is half a ball: a bowl. Volume = (2/3) π r^3. Its curved surface is 2 π r^2; a solid hemisphere also has a flat top, so total surface = 3 π r^2.



Sphere: volume = 4/3 π r^3, surface = 4 π r^2. Hemisphere: 2/3 π r^3, total surface 3 π r^2

`Sphere: volume (4/3) π r³, surface 4 π r²`	
`Hemisphere: volume (2/3) π r³, CSA 2 π r², TSA 3 π r²`	TSA adds the flat circle on top.

Example 1: Find the volume of a sphere of radius 21 cm (π = 22/7).

1. (4/3) × (22/7) × 21 × 21 × 21.
   Why: Formula for a sphere.
2. Cancel first: one 21 divided by 7 is 3, another 21 divided by 3 is 7. Left: 4 × 22 × 3 × 7 × 21.
   Why: Cancelling before multiplying keeps the numbers small.
3. 4 × 22 = 88 and 3 × 7 × 21 = 441, so 88 × 441 = 38,808 cm^3.

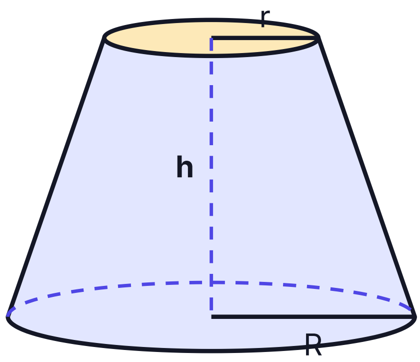
Answer: 38808

Frustum, prism and pyramid

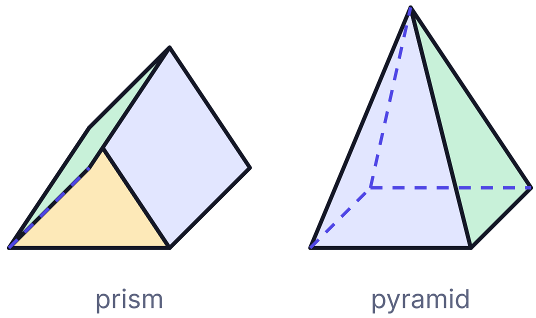
Cut the top off a cone with a slice parallel to the base and the bottom piece is a frustum: a bucket or a glass. With top radius r, bottom radius R and height h, volume = (1/3) π h (R^2 + r^2 + Rr).

A prism has the same shape all the way through (a triangular chocolate box, a pencil). Volume = area of the base shape × length.

A pyramid rises from a base shape to a point. Like a cone, it is one third of the matching prism: volume = (1/3) × base area × height.



Frustum (bucket): volume = 1/3 π h (R^2 + r^2 + Rr)



Prism: volume = base area × height. Pyramid: 1/3 × base area × height

`Frustum volume = (1/3) π h (R² + r² + Rr)`	Big cone minus small cone, done once.
`Prism volume = base area × height`	Any constant cross-section.
`Pyramid volume = (1/3) × base area × height`	One third of the prism.

Example 1: A bucket is a frustum with top radius 14 cm, bottom radius 7 cm and height 15 cm. Find its capacity in cm^3 (π = 22/7).

1. $R^2 + r^2 + Rr = 196 + 49 + 98 = 343$.
   Why: Work out the bracket first.
2. Volume = 1/3 × 22/7 × 15 × 343.
   Why: Put it into the formula.
3. = 22 × 5 × 49 = 5390 cm^3.
   Why: 15/3 = 5 and 343/7 = 49 cancel cleanly.

Answer: 5390

Example 2: A pyramid has a square base of side 6 m and height 10 m. Find its volume.

1. Base area = 36 m^2.
   Why: Square base.
2. Volume = 1/3 × 36 × 10 = 120 m^3.

Answer: 120

Scaling, and melting one solid into another

If every length of a shape grows by a factor k, its area grows by k^2 and its volume by k^3. Double the radius of a ball and it needs 4 times the paint but holds 8 times the air.

As percentages: if each side rises by x%, the area rises by $(2x + x^2/100)$%. A 10% longer side gives 21% more area and 33.1% more volume.

When one solid is melted and recast into another, nothing is lost: the volume stays the same. Write 'volume before = volume after', cancel π, and solve.

Lengths × k → area × k^2, volume × k^3	
Each side up x% → area up $(2x + x^2/100)$%	Successive percentage change applied twice.
Melt and recast: volume stays the same	Set the two volume formulas equal.
Number of small solids = big volume / small volume	

Example 1: A metal sphere of radius 6 cm is melted and drawn into a wire of radius 0.2 cm. Find the length of the wire in metres.

1. Sphere volume = $(4/3)\,\pi \times 216 = 288\,\pi$.
   Why: Keep π; it cancels.
2. Wire is a cylinder: $\pi \times 0.04 \times L$.
   Why: $r = 0.2$, so $r^2 = 0.04$.
3. $288\,\pi = 0.04\,\pi\,L$, so $L = 7200$ cm.
   Why: Volume is conserved; divide.
4. = 72 m.

Answer: 72

Example 2: Each side of a rectangle is increased by 20%. By what percent does its area increase?

1. Take sides 10 and 10: area 100. New sides 12 and 12: area 144.
   Why: The 'take 100' method from Easy Maths.
2. Increase = 44%. (Formula: 20 + 20 + 400/100 = 44.)

Answer: 44%

Exam tip: Melting questions: write both volume formulas side by side and cancel π and any 1/3 or 4/3 before multiplying anything.

Formula summary

Rectangle	Area lb, perimeter 2(l + b), diagonal $\sqrt{l^2 + b^2}$.
Triangle (Heron)	$s = (a + b + c)/2$, area $\sqrt{s(s - a)(s - b)(s - c)}$.
Equilateral	$\sqrt{3}/4\, a^2$.
Circle	πr^2, circumference $2\pi r$. Sector: (theta/360) πr^2.
Cube / cuboid	a^3, $6a^2$ / lbh, 2(lb + bh + hl). Cube diagonal $a\sqrt{3}$.
Cylinder	Volume $\pi r^2 h$, CSA $2\pi r h$, TSA $2\pi r(r + h)$.
Cone	Volume $(1/3)\pi r^2 h$, slant $l = \sqrt{r^2 + h^2}$, CSA $\pi r l$.
Sphere / hemisphere	$(4/3)\pi r^3$, $4\pi r^2$ / $(2/3)\pi r^3$, $3\pi r^2$.
% change of area	Each side changes x% → area changes $2x + x^2/100$ %.

Shortcuts

Melt and recast Use when: One solid melted into one or many others.

1. Volume is conserved: set the volume formulas equal.
2. Cancel π and fractions before multiplying.

Try it: A hemispherical bowl of radius 9 cm is full of liquid, poured into cylinders of radius 1.5 cm and height 4 cm. How many cylinders?

1. Bowl = (2/3) π × 729 = 486 π.
   Why: Hemisphere volume.
2. Cylinder = π × 2.25 × 4 = 9 π.
   Why: $\pi r^2 h$.
3. 486 / 9 = 54 cylinders.

Answer: 54

Spot the triple Use when: Any right triangle, cone slant, rhombus side or ladder question.

1. Check whether the numbers are a multiple of 3-4-5, 5-12-13, 8-15-17 or 7-24-25 before taking a square root.

Try it: A ladder 13 m long reaches a window 12 m high. How far is its foot from the wall?

1. 5, 12, 13 is a triple.
   Why: Ladder = hypotenuse, wall = one side.
2. 5 m.

Answer: 5

Percent change of area Use when: Sides change by x% and y%.

1. Change in area = x + y + xy/100 (use minus signs for decreases).

Try it: Length +20%, breadth -20%. Change in area?

1. 20 - 20 + (20 × -20)/100 = -4.
   Why: Successive-change formula.
2. 4% decrease.

Answer: -4%

Common mistakes

Wrong: Using the slanted side as the height of a parallelogram or trapezium.	**Instead:** Height is always the perpendicular distance between the parallel sides.
Wrong: Semicircle perimeter = π r only.	**Instead:** Add the diameter: π r + 2r.

Wrong: Using the vertical height for the cone's curved surface.	**Instead:** Surface uses the slant height $l = \sqrt{(r^2 + h^2)}$.
Wrong: Mixing cm and m in one calculation.	**Instead:** Convert everything to one unit before you start; 1 m^3 = 1000 litres.
Wrong: Thinking a 20% longer side gives 20% more area.	**Instead:** Area scales with the square: $1.2^2 = 1.44$, so 44% more.

Check yourself

- You can say whether a question wants perimeter, area or volume, and check the units in the options.
- You can name a triangle by its sides and by its angles, and find its area three ways.
- You can find the area of a parallelogram, rhombus and trapezium, and explain why each formula works.
- You can use sector = (angle/360) of the circle, and remember the diameter in a semicircle's perimeter.
- You can find volume and surface of a cuboid, cylinder, cone and sphere, and solve a melt-and-recast question.

Class plan (2 hours)

`10 min`	Speed maths warm-up: squares 11 to 25 round the class, then two squares of numbers ending in 5.
`10 min`	Hook: fence, tiles, water tank. Perimeter vs area vs volume with a grid on the board. Units check habit.
`15 min`	Triangles: draw the six types, let students name them. Angle sum. Area = 1/2 b h and why; Heron with 13-14-15; Pythagorean triples table.
`15 min`	Quadrilaterals: rectangle and square, then parallelogram (cut and slide), rhombus (diagonals), trapezium (two copies make a parallelogram).
`10 min`	Circles: circumference and area, sector as a fraction of 360, semicircle perimeter trap, ring. Wheel-turns example.
`5 min`	Polygons: triangles from one corner, angle sum, exterior angle trick, diagonals.
`15 min`	Solids: cuboid and cube, cylinder (unroll the label), cone (1/3 of a cylinder), sphere and hemisphere. Draw each once.
`5 min`	Frustum, prism, pyramid: one example each.
`10 min`	Scaling and melting: k^2 and k^3, the sphere-to-wire example.
`15 min`	Individual practice: easy and medium questions on the practice page, timed.
`5 min`	Go through two hard questions on the projector.
`5 min`	Recap and exit ticket: area of a trapezium with parallel sides 8 and 12, height 5 (answer 50).

Practice: 27 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/mensuration

Probability

> **If I toss a coin, will it be heads? Nobody knows. But everyone in this room would bet it is heads about half the time. Probability is simply that 'about half', written as a number.**

Placement tests (TCS, Infosys, Wipro, Accenture, AMCAT, CoCubes) ask 1 to 3 probability questions in almost every paper: coins, dice, cards, balls in a bag, and 'A and B try to solve a problem'. They look different but use the same four ideas: count the outcomes, use 'not' for 'at least one', add for OR, multiply for AND. Learn those four and this topic becomes free marks.

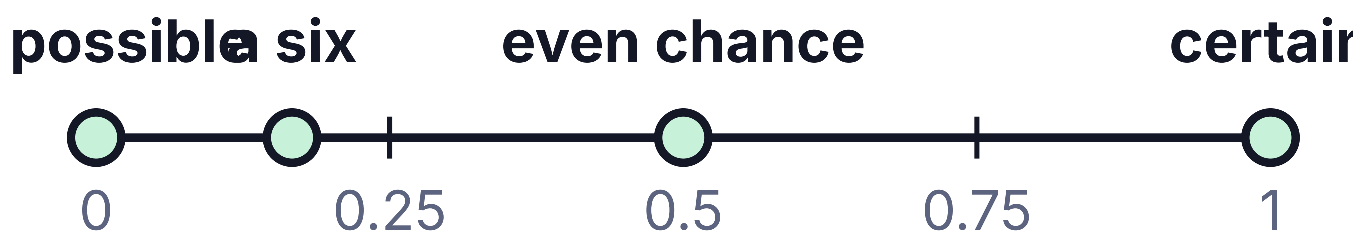
What probability means

Probability is a number between 0 and 1 that says how likely something is. 0 means it cannot happen. 1 means it is certain. 1/2 means it happens half the time.

To find it, count. Probability of an event = (number of ways the event can happen) / (total number of possible outcomes). This only works when every outcome is equally likely: a fair coin, a fair die, a well-shuffled deck, a ball drawn without looking.

Four words you will see in every question. Experiment: the action (toss a coin, draw a card). Outcome: one possible result (heads). Sample space: the list of ALL possible outcomes ({H, T}). Event: the outcomes we care about (getting heads).

Probabilities of all the outcomes add up to 1. That is why P(not E) = 1 - P(E): if there is a 1/6 chance of a six, there is a 5/6 chance of no six.



Every probability sits somewhere between 0 and 1

`P(E) = favourable outcomes / total outcomes`	Count what you want, count everything, divide.
`0 ≤ P(E) ≤ 1`	If you get more than 1 or a negative number, you have made a counting mistake.
`P(not E) = 1 - P(E)`	Something either happens or it does not.

Example 1: A bag has 3 red, 4 green and 5 blue balls. One ball is taken out without looking. What is the probability it is green?

1. Total outcomes = 3 + 4 + 5 = 12 balls.
   Why: Each ball is equally likely to be picked, so each ball is one outcome.
2. Favourable outcomes = 4 green balls.
   Why: These are the outcomes that make our event (green) happen.
3. P(green) = 4/12 = 1/3.

Answer: 1/3

Example 2: A number is chosen at random from 1 to 20. What is the probability it is a multiple of 3 or 5?

1. Multiples of 3: 3, 6, 9, 12, 15, 18 (6 numbers). Multiples of 5: 5, 10, 15, 20 (4 numbers).
   Why: List them; the list is short.
2. 15 is in both lists, so count it once: 6 + 4 - 1 = 9.
   Why: Counting 15 twice is the most common mistake in 'or' questions.
3. P = 9/20.

Answer: 9/20

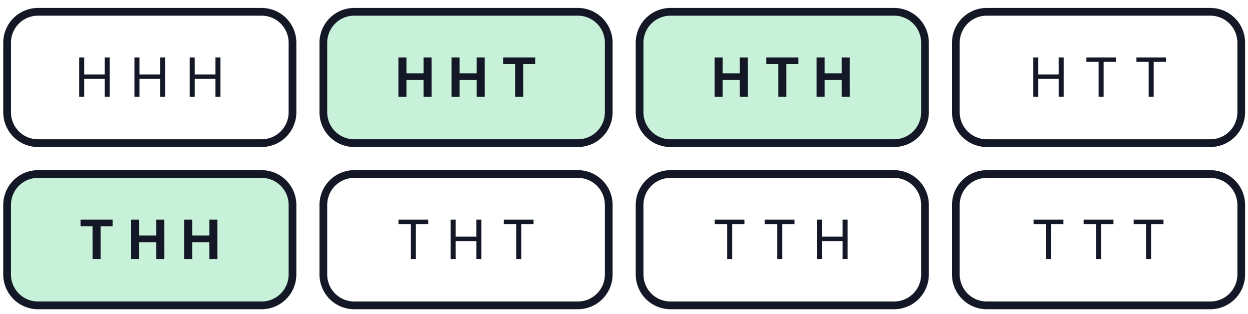
Exam tip: Before calculating, say the total out loud: '36 for two dice', '52 for cards', '8 for three coins'. Half of all wrong answers come from the wrong total.

Coins

One coin has 2 outcomes: H and T. Two coins have 2 × 2 = 4: HH, HT, TH, TT. Three coins have 2 × 2 × 2 = 8. In general n coins give 2^n outcomes.

HT and TH are DIFFERENT outcomes. Think of the coins as a 1-rupee and a 2-rupee coin: 'first heads, second tails' is not the same as 'first tails, second heads'. If you merge them you get 3 outcomes that are not equally likely, and every answer comes out wrong.

For 'exactly k heads', you do not need to list anything: the number of ways is nCk (choose which k coins show heads). So P(exactly k heads) = nCk / 2^n.



Three coins: 3 of the 8 outcomes have exactly two heads

`n coins: 2^n outcomes`	1 coin 2, 2 coins 4, 3 coins 8, 4 coins 16.
`P(exactly k heads) = nCk / 2^n`	Choose which k coins are heads.
`P(at least one head) = 1 - 1/2^n`	The only way to fail is all tails.

Example 1: Three coins are tossed. What is the probability of getting exactly two heads?

1. Total = 2^3 = 8 outcomes.
   Why: Each coin doubles the number of outcomes.
2. Exactly two heads: HHT, HTH, THH = 3 outcomes (or 3C2 = 3).
   Why: Choosing which 2 of the 3 coins are heads gives the count without listing.
3. P = 3/8.

Answer: 3/8

Example 2: Four coins are tossed. What is the probability of getting at least one tail?

1. Total = 2^4 = 16.
   Why: Four coins.
2. The opposite of 'at least one tail' is 'no tails', which is HHHH: only 1 outcome.
   Why: Counting the opposite is quicker than counting 1, 2, 3 and 4 tails separately.
3. P = 1 - 1/16 = 15/16.

Answer: 15/16

Exam tip: 'At least one' with coins is always 1 - $1/2^n$. Four coins: 15/16. Five coins: 31/32.

Dice

A die has 6 faces, so one die gives 6 outcomes. Two dice give 6 × 6 = 36 outcomes, written as pairs: (1,1), (1,2) ... (6,6). Draw the 6 × 6 grid below once in your notebook. Every two-dice question is just shading the right boxes and counting them.

Sums follow a pattern. Sum 2 has 1 way, sum 3 has 2 ways, and so on up to sum 7, which has 6 ways. Then it falls: sum 8 has 5 ways, sum 12 has 1. So the number of ways for a sum s is (s - 1) when s is 7 or less, and (13 - s) when s is more than 7.

Doublets (both dice the same) are the diagonal of the grid: 6 of them.

second die

first die	1	2	3	4	5	6
1	2	3	4	5	6	**7**
2	3	4	5	6	**7**	8
3	4	5	6	**7**	8	9
4	5	6	**7**	8	9	10
5	6	**7**	8	9	10	11
6	**7**	8	9	10	11	12

Sum 7: the 6 shaded boxes out of 36. P = 6/36 = 1/6

second die

first die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	**10**
5	6	7	8	9	**10**	**11**
6	7	8	9	**10**	**11**	**12**

Sum at least 10: 6 boxes. P = 6/36 = 1/6

Sum	2	3	4	5	6	7	8	9	10	11	12
Ways out of 36	1	2	3	4	5	6	5	4	3	2	1

Example 1: Two dice are thrown. What is the probability that the sum is 8?

1. Total = 36.
   Why: Two dice.
2. Sum 8: (2,6), (3,5), (4,4), (5,3), (6,2) = 5 ways.
   Why: Rule: for sums above 7, ways = 13 - sum = 13 - 8 = 5.
3. P = 5/36.

Answer: 5/36

Example 2: Two dice are thrown. What is the probability of getting a doublet or a total of 4?

1. Doublets: (1,1) ... (6,6) = 6 outcomes.
   Why: The diagonal of the grid.
2. Total 4: (1,3), (2,2), (3,1) = 3 outcomes.
   Why: Sum 4 has 4 - 1 = 3 ways.
3. (2,2) is in both lists, so the union is 6 + 3 - 1 = 8.
   Why: OR means 'either', and the overlap must only be counted once.
4. P = 8/36 = 2/9.

Answer: 2/9

Example 3: Three dice are thrown. What is the probability that all three show the same number?

1. Total = 6^3 = 216.
   Why: Each die multiplies the outcomes by 6.
2. All the same: (1,1,1), (2,2,2) ... (6,6,6) = 6 outcomes.
   Why: One outcome per number.
3. P = 6/216 = 1/36.

Answer: 1/36

Exam tip: Two dice, sum s: ways = 6 - |7 - s|. Sum 10 → 6 - 3 = 3 ways. You never need to list them.

Cards

A pack has 52 cards in 4 suits: spades and clubs (black), hearts and diamonds (red). Each suit has 13 cards: A, 2, 3 ... 10, J, Q, K.

So there are 4 of every rank (4 aces, 4 kings), 13 of every suit, and 26 of each colour. Face cards (also called court cards) are J, Q and K: 3 per suit, 12 in total.

When several cards are drawn together, count with combinations: the number of ways to draw 2 cards from 52 is 52C2 = 1326.

Spades	A	2	3	4	5	6	7	8	9	10	J	Q	K
Clubs	A	2	3	4	5	6	7	8	9	10	J	Q	K
Hearts	A	2	3	4	5	6	7	8	9	10	J	Q	K
Diamonds	A	2	3	4	5	6	7	8	9	10	J	Q	K

A deck: 52 cards = 4 suits × 13 ranks. 26 red, 26 black. 12 face cards (J, Q, K), 4 aces

Example 1: One card is drawn from a pack of 52. What is the probability that it is a king or a heart?

1. Kings: 4. Hearts: 13.
   Why: Count each group.
2. The king of hearts is in both groups, so kings or hearts = 4 + 13 - 1 = 16.
   Why: OR: add, then subtract the overlap.
3. P = 16/52 = 4/13.

Answer: 4/13

Example 2: Two cards are drawn together from a pack of 52. What is the probability that both are red?

1. Ways to draw any 2 cards: 52C2 = 52 × 51 / 2 = 1326.
   Why: 'Drawn together' means order does not matter, so use combinations.
2. Ways to draw 2 red cards: 26C2 = 26 × 25 / 2 = 325.
   Why: Choose both from the 26 red cards.
3. P = 325/1326 = 25/102.

Answer: 25/102

Example 3: One card is drawn. What is the probability that it is a face card that is not red?

1. Face cards: J, Q, K in each of 4 suits = 12.
   Why: 3 per suit.
2. Not red means black: spades or clubs, so 2 suits × 3 = 6.
   Why: Half of the face cards.
3. P = 6/52 = 3/26.

Answer: 3/26

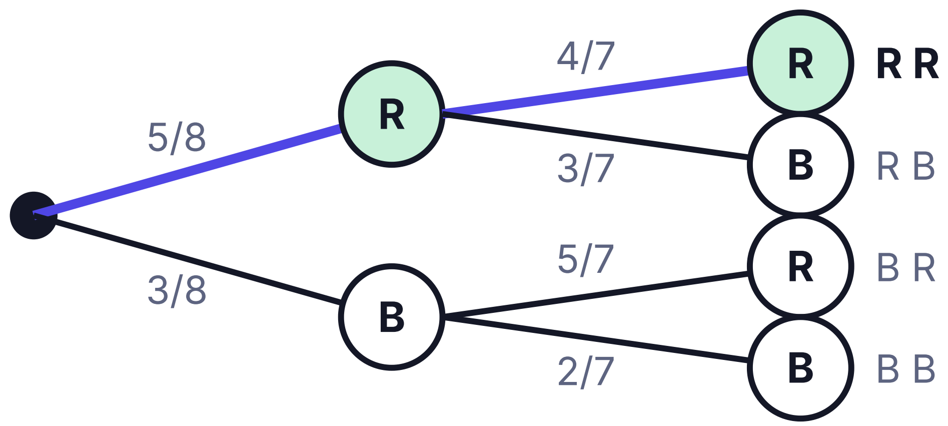
Exam tip: Write '52 = 4 × 13, 26 red, 12 face, 4 of each' at the top of your rough sheet before a card question.

Balls in a bag: with and without replacement

'With replacement' means the ball goes back before the next draw, so every draw looks exactly like the first. 'Without replacement' means it stays out, so the second draw has one ball fewer, and one fewer of whatever colour came out.

If balls are drawn 'together' or 'at the same time', that is the same as without replacement, and combinations are the fastest way to count.

Two ways to solve a without-replacement question, and both give the same answer: multiply step by step (5/8 × 4/7), or count with combinations (5C2 / 8C2). Use whichever you find easier.



Without replacement: the second branch depends on the first draw

Example 1: A bag has 5 red and 3 blue balls. Two balls are drawn without replacement. What is the probability both are red?

1. First draw red: 5/8.
   Why: 5 red out of 8 balls.
2. Second draw red, given the first was red: 4/7.
   Why: One red ball is already out, so 4 red remain out of 7.
3. Both: 5/8 × 4/7 = 20/56 = 5/14.
   Why: AND means multiply.
4. Check with combinations: 5C2 / 8C2 = 10/28 = 5/14. Same.

Answer: 5/14

Example 2: Same bag, but the first ball is put back before the second draw. Probability both are red?

1. First red: 5/8. Second red: 5/8 again.
   Why: The ball went back, so the bag is unchanged.
2. P = 5/8 × 5/8 = 25/64.

Answer: 25/64

Example 3: A bag has 3 red and 5 black balls. Two are drawn together. What is the probability of getting one of each colour?

1. Total ways: 8C2 = 28.
   Why: Drawn together, so combinations.
2. One red AND one black: 3C1 × 5C1 = 3 × 5 = 15.
   Why: Choose the red one (3 ways) and the black one (5 ways); multiply.
3. P = 15/28.

Answer: 15/28

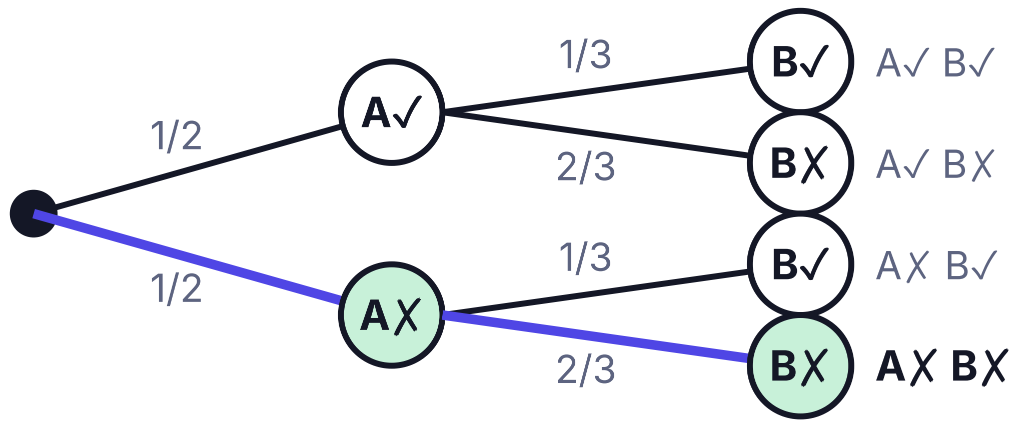
Exam tip: 'Together' = combinations. 'One after another without replacement' = multiply fractions that shrink by one.

At least one: use the complement

'At least one' means one, or two, or three, or more. Counting all of those cases separately is slow and easy to get wrong.

The opposite of 'at least one' is 'none'. And 'none' is usually one simple case. So P(at least one) = 1 - P(none).

This is exactly how to solve 'A and B try to solve a problem' questions. The problem gets solved if at least one of them solves it. So find the probability that NEITHER solves it, and subtract from 1.



Only the A✗ B✗ path leaves the problem unsolved: 1/2 × 2/3 = 1/3

`P(at least one) = 1 - P(none)`	Works for coins, dice, people, machines, shots at a target.
`P(none) for independent events = (1 - p1)(1 - p2)(1 - p3)...`	Each person fails independently, so multiply the failure chances.

Example 1: A and B solve a problem independently with probabilities 1/2 and 1/3. What is the probability that the problem gets solved?

1. 'The problem gets solved' means at least one of A and B solves it.
   Why: Rewrite the question in these words first; it tells you which rule to use.
2. P(A fails) = 1 - 1/2 = 1/2. P(B fails) = 1 - 1/3 = 2/3.
   Why: Failure is the complement of success for each person.
3. P(both fail) = 1/2 × 2/3 = 1/3.
   Why: They work independently, so multiply.
4. P(solved) = 1 - 1/3 = 2/3.
   Why: Solved is the opposite of 'both fail'.
5. Check: 1/2 + 1/3 would be 5/6, which counts the case where both solve it twice. That is why 5/6 is the trap option.

Answer: 2/3

Example 2: A die is thrown 3 times. What is the probability of getting at least one six?

1. P(no six on one throw) = 5/6.
   Why: 5 of the 6 faces are not a six.
2. P(no six in 3 throws) = $(5/6)^3$ = 125/216.
   Why: The throws are independent, so multiply.
3. P(at least one six) = 1 - 125/216 = 91/216.

Answer: 91/216

Example 3: Three students solve a problem with probabilities 1/2, 1/3 and 1/4, independently. What is the probability that it is solved?

1. Failure chances: 1/2, 2/3, 3/4.
   Why: 1 minus each success chance.
2. All three fail: 1/2 × 2/3 × 3/4 = 6/24 = 1/4.
   Why: Independent, so multiply.
3. P(solved) = 1 - 1/4 = 3/4.

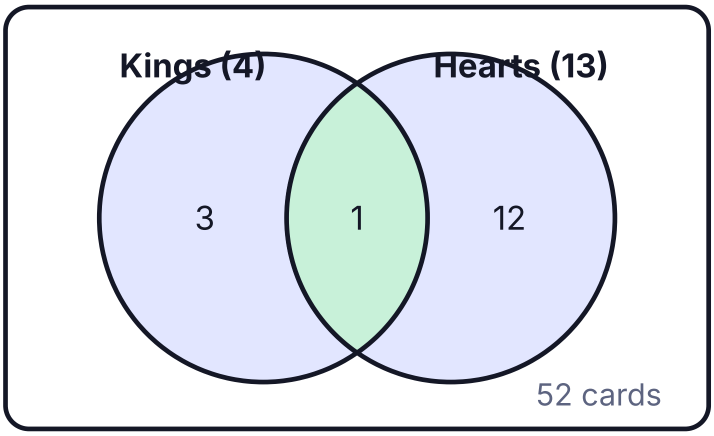
Answer: 3/4

Exam tip: Read 'at least', 'not all', 'the problem is solved', 'the target is hit' as: 1 minus P(nobody).

OR: the addition rule

P(A or B) means the chance that A happens, or B happens, or both. If you just add P(A) + P(B), the outcomes where both happen get counted twice. So subtract them once: P(A or B) = P(A) + P(B) - P(A and B).

If A and B can never happen together (mutually exclusive), the overlap is empty and you simply add. Example: one die shows 2 or shows 5. It cannot show both, so 1/6 + 1/6 = 1/3.



King or heart: 4 + 13 - 1 = 16 cards

`P(A or B) = P(A) + P(B) - P(A and B)`	Add, then remove the double-counted overlap.
`Mutually exclusive: P(A or B) = P(A) + P(B)`	No overlap, nothing to remove.

Example 1: From the numbers 1 to 30, one is picked at random. What is the probability it is divisible by 2 or 3?

1. Divisible by 2: 15 numbers. By 3: 10 numbers.
   Why: 30/2 and 30/3.
2. Divisible by both means divisible by 6: 5 numbers.
   Why: The overlap of multiples of 2 and 3 is multiples of 6.
3. 15 + 10 - 5 = 20. P = 20/30 = 2/3.

Answer: 2/3

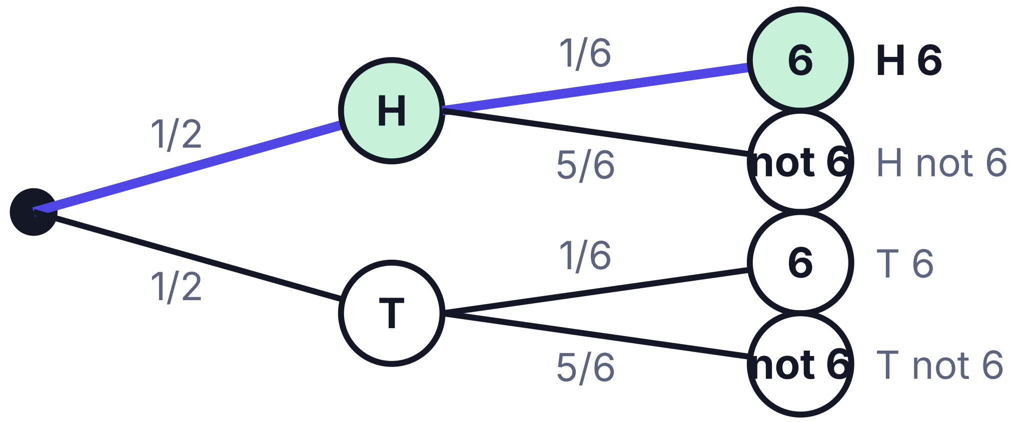
Exam tip: See the word OR? Add, then ask 'can both happen at once?' If yes, subtract the overlap.

AND: the multiplication rule

P(A and B) is the chance that both happen. If the two events do not affect each other (independent), multiply: P(A) × P(B). Coins, dice and draws WITH replacement are independent.

If the first event changes the second (dependent), multiply by the second probability AFTER the first has happened. Draws WITHOUT replacement are dependent.

A probability tree shows this: multiply along a branch to get AND, add different branches to get OR.



Coin then die: heads AND six = 1/2 × 1/6 = 1/12

Example 1: A coin is tossed and a die is thrown. What is the probability of heads and an even number?

1. P(heads) = 1/2. P(even) = 3/6 = 1/2.
   Why: Three even faces: 2, 4, 6.
2. Independent, so P = 1/2 × 1/2 = 1/4.
   Why: The coin does not affect the die.

Answer: 1/4

Example 2: Two cards are drawn one after the other without replacement. What is the probability that both are aces?

1. First ace: 4/52.
   Why: 4 aces in 52 cards.
2. Second ace: 3/51.
   Why: One ace and one card are gone.
3. P = 4/52 × 3/51 = 12/2652 = 1/221.

Answer: 1/221

Example 3: A speaks the truth 3/4 of the time and B 4/5 of the time. What is the probability that they contradict each other on the same statement?

1. They contradict when exactly one of them tells the truth.
   Why: If both tell the truth or both lie, they agree.
2. A true and B lies: 3/4 × 1/5 = 3/20.
   Why: AND, independent: multiply.
3. A lies and B true: 1/4 × 4/5 = 4/20.
   Why: The other way round.
4. Add the two cases: 3/20 + 4/20 = 7/20.
   Why: The two cases cannot happen together, so plain addition.

Answer: 7/20

Exam tip: Say it in words: 'this AND that' → multiply. 'this case OR that case' → add the cases.

Conditional probability: 'given that'

Sometimes the question tells you something has already happened: 'given that the sum is even', 'if it is known that one child is a boy'. That information shrinks the sample space. Throw away every outcome that does not fit, and count only among the ones left.

As a formula: P(A given B) = P(A and B) / P(B). But the shrink-and-count method is easier to understand and gives the same answer.

second die

first die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	**8**
3	4	5	6	7	**8**	9
4	5	6	7	**8**	9	10
5	6	7	**8**	9	10	11
6	7	**8**	9	10	11	12

Given the sum is even (18 boxes), sum 8 is 5 of them: 5/18

`P(A given B) = P(A and B) / P(B)`	Among the outcomes where B happened, what fraction also have A?

Example 1: Two dice are thrown. Given that the sum is even, what is the probability that the sum is 8?

1. Outcomes with an even sum: 18 of the 36.
   Why: Half the grid; this is our new, smaller sample space.
2. Among those, sum 8: (2,6), (3,5), (4,4), (5,3), (6,2) = 5.
   Why: All of them have an even sum, so they are all inside the new space.
3. P = 5/18.

Answer: 5/18

Example 2: A family has two children. Given that at least one is a boy, what is the probability that both are boys?

1. All families: BB, BG, GB, GG.
   Why: Older child first; BG and GB are different families.
2. 'At least one boy' removes GG. Left: BB, BG, GB.
   Why: The given information shrinks the sample space to 3.
3. Both boys = BB, 1 of 3. P = 1/3.

Answer: 1/3

Exam tip: 'Given that' → cross out every outcome that breaks the given fact, then count again.

Odds, and how questions are worded

Odds are another way of saying probability. 'Odds in favour of E are 3 : 5' means for every 3 ways E happens there are 5 ways it does not. So there are 8 equally likely ways in total and P(E) = 3/8. 'Odds against E are 3 : 5' means P(E) = 5/8.

The table below turns the usual phrases into the rule you need.

The question says	It means	Use
at least one	one or more	1 - P(none)
at most one	zero or one	P(0) + P(1)
exactly one	one and only one	add the cases (first only, second only...)
neither	none of them	multiply the failure chances
either ... or	OR	addition rule
both / and	AND	multiplication rule
given that / if it is known	conditional	shrink the sample space
drawn together / at random at once	no order	combinations nC r
odds in favour a : b	P = a/(a + b)	

Example 1: The odds against an event are 7 : 2. What is its probability?

1. Odds against 7 : 2 means 7 ways it fails for every 2 ways it happens.
   Why: Against puts failure first.
2. Total 9 ways. P(event) = 2/9.

Answer: 2/9

Formula summary

`P(E) = favourable / total`	
`P(not E) = 1 - P(E)`	'At least one' is almost always 1 - P(none).
`Addition`	P(A or B) = P(A) + P(B) - P(A and B).
`Independent`	P(A and B) = P(A) × P(B).
`Conditional`	P(A | B) = P(A and B) / P(B).
`Two dice`	36 outcomes. Sum 7 has 6 ways, the most of any sum.
`Cards`	52 = 4 suits × 13. 12 face cards. 4 of each rank.

Shortcuts

At least one = 1 minus none Use when: 'At least one six', 'the problem is solved', 'the target is hit'.

1. Find the chance that it fails every time.
2. Subtract from 1.

Try it: At least one six in three throws of a die?

1. P(no six) in one throw = 5/6, so in three throws $(5/6)^3$ = 125/216.
   Why: Independent throws multiply.
2. 1 - 125/216 = 91/216.

Answer: 91/216

Sum of two dice without listing Use when: Any 'sum is s' question with two dice.

1. Ways = 6 - |7 - s|.
2. Divide by 36.

Try it: Sum of 10 with two dice?

1. 6 - |7 - 10| = 6 - 3 = 3 ways: (4,6), (5,5), (6,4).
   Why: Sums are symmetric around 7.
2. 3/36 = 1/12.

Answer: 1/12

Exactly k heads Use when: Any coin question asking for an exact count.

1. Ways = nCk. Total = 2^n.

Try it: Exactly 3 heads in 5 tosses?

1. 5C3 = 10 ways.
   Why: Choose which 3 tosses are heads.
2. Total 2^5 = 32. P = 10/32 = 5/16.

Answer: 5/16

Common mistakes

Wrong: Treating HT and TH as the same outcome, so two coins seem to have 3 outcomes.	**Instead:** Label the coins (first, second). Two coins always have 4 outcomes.
Wrong: Adding success chances: 1/2 + 1/3 = 5/6 for 'A or B solves it'.	**Instead:** Use 1 - P(both fail) = 1 - 1/2 × 2/3 = 2/3.
Wrong: Keeping the bag the same size on the second draw without replacement.	**Instead:** Take one ball out of the total and out of its colour: 5/8 then 4/7.
Wrong: Counting the king of hearts twice in 'king or heart'.	**Instead:** OR: add, then subtract what is in both groups.
Wrong: Forgetting that 'drawn together' is a combination question.	**Instead:** Use nCr for both the favourable and the total count.

Check yourself

- You can say what total to use for coins (2^n), dice (6^n) and cards (52), before calculating.
- You can turn 'at least one' into 1 - P(none) and solve the 'A and B solve a problem' question.
- You can add for OR (and subtract the overlap), multiply for AND.
- You can tell with replacement from without, and adjust the second fraction.
- You can shrink the sample space for 'given that' questions.

Class plan (2 hours)

`10 min`	Speed maths warm-up: five 2-digit criss-cross multiplications on the board, students race. (Keeps the daily habit.)
`10 min`	Hook: the coin question. Build the 0-to-1 line together; students place 'rain tomorrow', 'sun rises', 'a six' on it. Define outcome, sample space, event.
`15 min`	Coins and dice. Draw the 6 × 6 dice grid on the board; students shade sum 7, sum 8, doublets and count. Show HT and TH are different.
`10 min`	Cards: 52 = 4 × 13. Students fill the deck breakdown from memory, then the king-or-heart example.

`15 min`	At least one = 1 - none. Do the A and B problem (the one students got wrong in the test) with the tree, then the three-students version.
`15 min`	OR and AND. Venn diagram for the addition rule, tree for the multiplication rule. With vs without replacement with the bag example.
`10 min`	Conditional: shrink the sample space. Dice given even sum; the two-children puzzle (let the class argue 1/2 vs 1/3 first).
`20 min`	Individual practice: the easy and medium questions on the practice page, timed. Walk the room.
`10 min`	Go through the three hard questions together on the projector (present mode).
`5 min`	Recap: each student says one rule in their own words. Exit ticket: at least one six in two throws (answer 11/36).

Practice: 24 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/probability

Permutation and Combination

> **How many ways can 3 people stand in a line? Try it: 6. How many ways to pick 2 of them for a team? 3. Arranging and choosing are different questions.**

Counting is the engine of probability too. The whole topic: multiply for 'and', add for 'or', use P when order matters and C when it does not.

The counting principle: boxes to fill

If one choice can be made in m ways and the next in n ways, together they can be made in m × n ways (AND means multiply).

Draw a box for each position and write the number of choices in it. Multiply.

President, secretary, treasurer from 5 people: 5 × 4 × 3 = 60 ways

Example 1: How many 3-digit numbers can be formed with the digits 1 to 6 if no digit repeats?

1. Hundreds: 6 choices. Tens: 5. Units: 4.
   Why: Each used digit is no longer available.
2. 6 × 5 × 4 = 120.

Answer: 120

Example 2: How many 3-digit numbers with digits from 0 to 9, repetition allowed?

1. Hundreds: 9 choices (no 0 in front). Tens: 10. Units: 10.
   Why: A number cannot start with 0.
2. 900.

Answer: 900

Permutations: order matters

nPr = n! / (n - r)! = n × (n - 1) x ... (r numbers). Arranging all n things: n!.

Identical items: divide by the factorial of each repeat. Arrangements of BANANA = 6! / (3! 2!) = 60.

Items that must stay together: glue them into one block, arrange, then arrange inside the block.

Example 1: In how many ways can the letters of LEADER be arranged?

1. 6 letters, E twice.
   Why: Repeated letters make some arrangements identical.
2. 6! / 2! = 360.

Answer: 360

Example 2: In how many arrangements of the word APPLE do the two Ps come together?

1. Glue PP into one block: A, PP, L, E = 4 items.
   Why: Together means one block.
2. 4! = 24.
   Why: The two Ps are identical, so no inside arrangement.
3. 24.

Answer: 24

Combinations: order does not matter

nCr = n! / (r! (n - r)!) = nPr / r!. Teams, committees, handshakes, groups: choosing, not arranging.

nCr = nC(n - r): choosing 3 to play is the same as choosing 7 to sit out of 10.

Example 1: A committee of 3 men and 2 women is to be chosen from 6 men and 5 women. How many ways?

1. Men: 6C3 = 20. Women: 5C2 = 10.
   Why: Choose each group separately.
2. 20 × 10 = 200.

Answer: 200

Example 2: At least one woman: a team of 3 from 4 men and 3 women.

1. All teams: 7C3 = 35.
2. No women: 4C3 = 4.
   Why: 'At least one' → total minus none.
3. 35 - 4 = 31.

Answer: 31

Circles and restrictions

n people around a table: (n - 1)!. Fix one person; arrange the rest. Necklaces and garlands that can be flipped: (n - 1)!/2.

Two people must NOT sit together: total - (arrangements where they are together).

Example 1: In how many ways can 6 people sit around a round table?

1. Fix one person. Arrange 5 others.
   Why: Rotations of the same seating count once.
2. 5! = 120.

Answer: 120

Example 2: 5 people in a row. Two of them, A and B, refuse to sit together. How many ways?

1. All: 5! = 120.
2. A and B together: glue them, 4! x 2! = 48.
   Why: The block can be AB or BA.
3. 120 - 48 = 72.

Answer: 72

Formula summary

`Counting principle`	Independent steps multiply (AND); alternatives add (OR).
`nPr = n!/(n - r)!`	Arrangements.
`nCr = n!/(r!(n - r)!)`	Selections.
`Repeated letters`	n!/(p! q! ...).
`Circular`	(n - 1)!. Necklace / garland (can flip): (n - 1)!/2.
`Items together`	Glue them into one block, arrange, then arrange inside the block.
`At least one of n items`	2^n - 1.
`Handshakes / lines`	nC2. Diagonals of an n-gon: nC2 - n.

Shortcuts

Glue the block Use when: 'Vowels together'.

1. ORANGE: vowels O, A, E become one block.
2. Arrange 4 things: 4!. Inside the block: 3!.
3. 24 × 6 = 144.

Try it: Arrangements of ORANGE with vowels together

1. 144.

Answer: 144

Common mistakes

Wrong: Using P for a committee.	**Instead:** Committees have no order: use C.

Wrong: Forgetting that a number cannot start with 0.	**Instead:** Count the first box separately.

Check yourself

- You can draw boxes and multiply.
- You can choose P or C by asking 'does order matter?'.
- You can handle repeats, blocks, circles and 'not together'.

Class plan (2 hours)

`10 min`	Warm-up: factorials cheat sheet.
`20 min`	Boxes: the counting principle.
`25 min`	Permutations, repeats, glue blocks.
`25 min`	Combinations, committees, 'at least'.
`15 min`	Circles and 'not together'.
`20 min`	Practice set.
`5 min`	Recap.

Practice: 13 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/permutation-and-combination

Data Interpretation

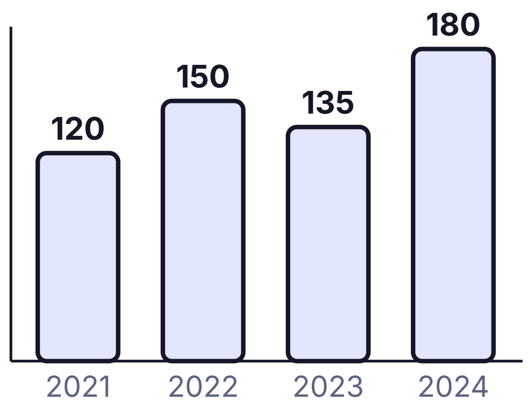
> **A chart is a question you answer by reading carefully. Half the marks in DI are lost by reading the wrong year, the wrong unit or the wrong bar.**

DI sets carry 4 or 5 questions on one table or chart, so one set is worth a lot of marks. The maths is percentages, ratios and averages; the skill is reading.

Read before you calculate

Read the title, the units (thousands? lakhs? %?), the axes and the legend before the first question.

Then approximate: DI options are usually far enough apart that rounding to 2 significant figures is enough.



Sales of a company (Rs crore)

Example 1: From the chart: by what percent did sales grow from 2021 to 2024?

1. Change = 180 - 120 = 60.
   Why: Read the two bars.
2. 60 / 120 × 100.
   Why: Percentage change is over the earlier year.
3. 50%.

Answer: 50%

Example 2: Average sales over the four years?

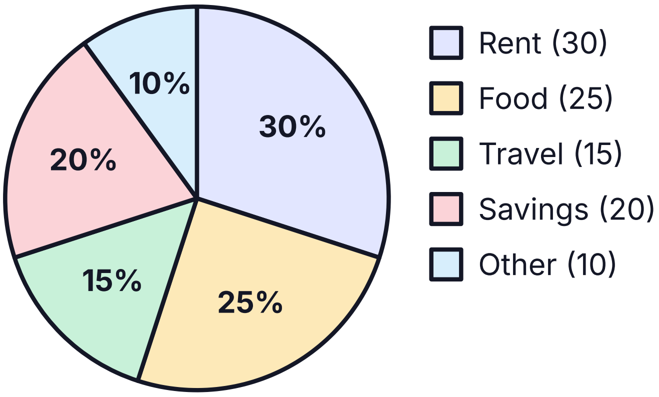
1. 120 + 150 + 135 + 180 = 585.
2. 585 / 4 = 146.25 crore.

Answer: 146.25

Pie charts

A pie chart shows shares of a whole. Sector angle = share × 360. A 90 degree sector is a quarter.

To get the actual value: share % × total.



Monthly spending of a family (%)

Example 1: If the family earns Rs 60,000 a month, how much more is spent on rent than on travel?

1. Rent - travel = 30% - 15% = 15%.
   Why: Work with the percentages first; multiply once.
2. 15% of 60,000 = 9,000.

Answer: 9000

Example 2: What is the angle of the Food sector?

1. 25% of 360.
2. = 90 degrees.

Answer: 90

Tables and line charts

For a table, circle the row and column you need before calculating. For a line chart, the steepest segment is the biggest change, and a fall shows a line going down.

Ratios across years: compare using fractions of the same base, not by computing each value.

Formula summary

`% growth`	(new - old)/old × 100.
`Share of total`	part/total × 100. For a pie: part/total × 360 degrees.
`Average across rows`	Total / count.
`Ratio`	Reduce by HCF; compare by cross-multiplying, not by dividing.

Shortcuts

Growth % by fractions Use when: Old 120, new 150.

1. 30/120 = 1/4 = 25%, no division needed.

Try it: 120 → 150

1. 1/4.

Answer: 25%

Check yourself

- You can read units and legends first.
- You can compute percentage change over the earlier value.
- You can turn pie percentages into values and angles.

Class plan (2 hours)

`10 min`	Warm-up: percentage table.
`20 min`	Reading habits: title, units, legend.
`25 min`	Bar chart set.
`25 min`	Pie chart set with angles.
`20 min`	Table and line chart sets.
`15 min`	Timed DI set.
`5 min`	Recap.

Practice: 8 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/data-interpretation

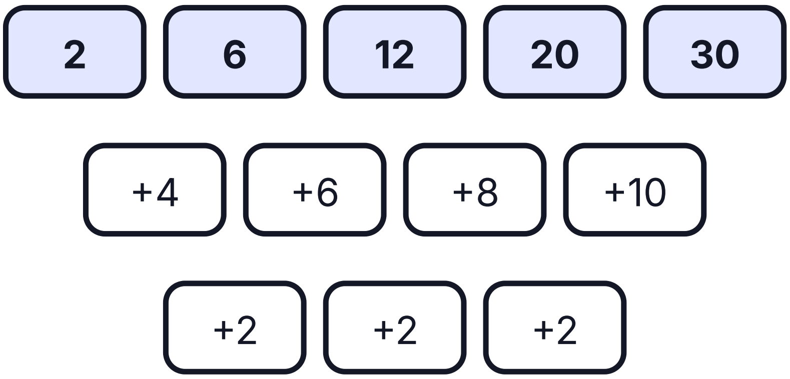
Series

> **2, 6, 12, 20, 30, ? Write the gaps underneath: 4, 6, 8, 10. The next gap is 12, so the answer is 42. Most series give up their secret in the gaps.**

Number and letter series are among the fastest marks in reasoning, if you have a fixed checklist. Without one, students stare at the numbers.

The checklist for number series

1. Differences: write the gaps. If the gaps are not constant, write the gaps of the gaps.
2. Ratios: if numbers grow fast, divide each by the one before (x 2, x 3, or × 2 + 1 ...).
3. Squares and cubes: look for n^2, n^3, $n^2 + 1$, $n^3 - 1$.
4. Two series mixed: odd positions follow one rule, even positions another.
5. Primes: 2, 3, 5, 7, 11, 13 ... hidden in the numbers or the gaps.



Gaps 4, 6, 8, 10 grow by 2. Next gap 12, next term 42

Example 1: 3, 7, 15, 31, 63, ?

1. Gaps: 4, 8, 16, 32. They double.
   Why: Step 1 of the checklist.
2. Also each term = previous × 2 + 1.
   Why: Two ways to see it; both agree.
3. 63 × 2 + 1 = 127.

Answer: 127

Example 2: 1, 8, 27, 64, ?, 216

1. These are 1^3, 2^3, 3^3, 4^3.
   Why: Step 3: cubes.
2. 5^3 = 125.

Answer: 125

Example 3: 4, 5, 12, 10, 36, 15, ?

1. Odd places: 4, 12, 36: x 3 each time.
   Why: Step 4: two series woven together.
2. Even places: 5, 10, 15: + 5.
3. Next is odd place: 36 × 3 = 108.

Answer: 108

Wrong number in a series

Find the rule from most of the terms, then check each term. The one that breaks the rule is wrong. Check the terms around it too: sometimes fixing one term fixes two gaps.

Example 1: Find the wrong number: 2, 5, 10, 17, 26, 38, 50

1. Gaps: 3, 5, 7, 9, 12, 12.
2. Should be 3, 5, 7, 9, 11, 13 (odd numbers).
   Why: Most gaps follow odd numbers.
3. 26 + 11 = 37, so 38 is wrong (and 37 + 13 = 50 confirms it).

Answer: 38

Letter series

Turn letters into numbers (A = 1 ... Z = 26) and treat it as a number series. Write the alphabet with positions at the top of your sheet.

Remember EJOTY: E = 5, J = 10, O = 15, T = 20, Y = 25. They let you find any letter's position in two steps.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Letter positions: A=1 ... Z=26 (reverse position underneath). Opposite pairs add to 27

Example 1: B, E, I, N, ?

1. Positions 2, 5, 9, 14.
   Why: Letters to numbers.
2. Gaps 3, 4, 5, so next gap 6.
3. 14 + 6 = 20 = T.

Answer: T

Example 2: AZ, BY, CX, ?

1. First letters A, B, C go forward. Second letters Z, Y, X go backward.
   Why: Each letter pair is an opposite pair (adds to 27).
2. DW.

Answer: DW

Formula summary

`Differences`	Write the gaps. Constant → AP. Gaps in AP → second difference constant.
`Ratios`	x2, x3, or x2 +1 style (multiply then add).
`Squares / cubes`	n^2 +/- 1, n^3 +/- n. Memorise squares to 30 and cubes to 15.
`Alternate series`	Odd positions and even positions follow two different rules.
`Letter series`	Convert to positions (A = 1 ... Z = 26). EJOTY = 5, 10, 15, 20, 25.
`Wrong number`	Find the rule from the majority, then the odd term out.

Shortcuts

Letter positions by anchors Use when: Converting letters fast.

1. EJOTY: E5 J10 O15 T20 Y25.
2. Reverse position = 27 - position (A <→ Z, M <→ N).

Try it: Next: B, E, J, Q, ?

1. 2, 5, 10, 17 = $n^2 + 1$.
2. Next 26 → Z.

Answer: Z

Check yourself

- You can run the five-step checklist.
- You can write gaps and gaps of gaps.
- You can convert letters to positions with EJOTY.

Class plan (2 hours)

`10 min`	Warm-up: squares and cubes quiz.
`25 min`	The five-step checklist with the gap ladder.
`15 min`	Two woven series.
`15 min`	Wrong number.

`20 min`	Letter series, EJOTY, opposite letters.
`30 min`	Practice set, timed.
`5 min`	Recap.

Practice: 12 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/series

Coding and Decoding

> **If CAT is written as DBU, each letter moved one step forward. Coding questions hide a simple rule like that; your job is to find it and apply it.**

Coding appears in almost every reasoning section. Most codes are one of four kinds: shift, reverse, opposite letter, or position number.

The four common codes

Shift: every letter moves the same number of steps (CAT → DBU is +1). The shift can also change per letter (+1, +2, +3 ...).

Reverse: the word is written backwards, sometimes with a shift too.

Opposite: each letter becomes its mirror in the alphabet (A <→ Z, B <→ Y). Pairs always add to 27.

Numbers: letters become their positions (CAT → 3 1 20), or the sum of positions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Letter positions: A=1 ... Z=26 (reverse position underneath). Opposite pairs add to 27

Example 1: If MOTHER is coded as NPUIFS, how is FATHER coded?

1. M → N, O → P, T → U: each letter +1.
   Why: Compare letter by letter.
2. FATHER +1 = GBUIFS.

Answer: GBUIFS

Example 2: If GOLD is coded as TLOW, how is SILK coded?

1. G (7) → T (20): 7 + 20 = 27. O → L: 15 + 12 = 27.
   Why: Opposite letters add to 27.
2. SILK → HROP.

Answer: HROP

Example 3: If CAT = 24, what is DOG?

1. C + A + T = 3 + 1 + 20 = 24.
   Why: Sum of positions.
2. D + O + G = 4 + 15 + 7 = 26.

Answer: 26

Sentence codes ('Chinese' coding)

You get several sentences and their codes. Words common to two sentences share a common code. Compare sentences in pairs and cross out what matches.

Example 1: 'sky is blue' = 'ta na pa', 'blue is deep' = 'na ka ta', 'sky and sea' = 'pa ro mi'. What is the code for 'deep'?

1. Sentences 1 and 2 share 'is' and 'blue', and codes 'ta' and 'na'.
   Why: Common words match common codes.
2. Sentence 2's remaining word 'deep' matches its remaining code.
3. 'ka'.

Answer: ka

Formula summary

`Shift code`	Each letter moves +k (or -k). CAT → DBU is +1.
`Reverse alphabet`	A <→ Z: position becomes 27 - position.
`Number code`	Letters to positions, sometimes summed or multiplied.
`Word substitution`	'Sky is called sea' → answer with the NEW name.
`Chinese / sentence code`	Compare two sentences: the common word and the common code match.

Shortcuts

Sentence code by overlap Use when: 'pit na sa' = you are good, 'na ho pit' = are you fine ...

1. Words common to two sentences map to codes common to the same two sentences.
2. Peel them off one at a time.

Try it: 'la pe ro' = sweet hot coffee, 'ro ki ne' = coffee is good, 'pe ne ta' = hot good tea. Code for 'good'?

1. 'coffee' is in sentences 1 and 2; the only code in both is 'ro'.
2. 'good' is in 2 and 3; the only code in both is 'ne'.

Answer: ne

Check yourself

- You can test shift, reverse, opposite and number codes in that order.
- You can use 27 to find opposite letters.
- You can match common words to common codes.

Class plan (2 hours)

`10 min`	Warm-up: letter positions with EJOTY.

`30 min`	Shift, reverse, opposite, numbers: one example each, then mixed.
`20 min`	Sentence codes.
`15 min`	Substitution codes ('sky is called water').
`40 min`	Practice set.
`5 min`	Recap.

Practice: 7 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/coding-and-decoding

Directions

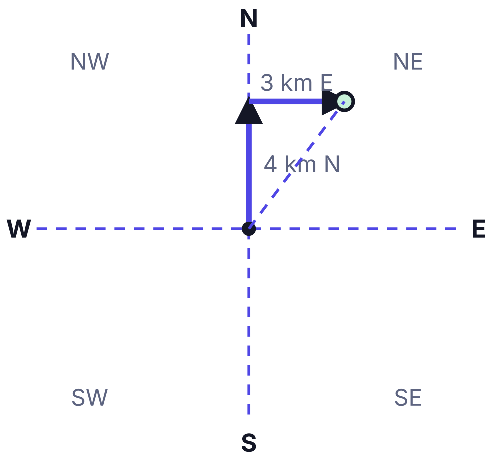
> **Draw it. Every direction question is easy on paper and hard in your head.**

Directions questions ask where someone ends up, how far from the start, or which way they face. A compass, a few arrows and Pythagoras solve all of them.

Turns and the compass

North is up, East is right. Facing north, a right turn faces east, a left turn faces west. Right turn = clockwise.

Keep track of which way the person faces after each turn, then draw that leg.



4 km north, then right 3 km: 5 km from the start, north-east

Example 1: Ravi walks 4 km north, turns right and walks 3 km. How far is he from the start?

1. After the right turn he faces east.
   Why: Right from north is east.
2. He is 4 km north and 3 km east of the start.
   Why: Draw it.
3. $\sqrt{16 + 9} = 5$ km.

Answer: 5

Example 2: A man walks 10 m south, turns left and walks 6 m, turns left and walks 10 m. Where is he from the start?

1. South 10, then left from south = east 6, then left from east = north 10.
   Why: Track the facing direction.
2. The south and north legs cancel: 6 m east of the start.

Answer: 6 m east

Shadows

In the morning the sun is in the east, so shadows fall to the west. In the evening shadows fall to the east. At noon there is almost no shadow.

If a person's shadow falls to his right in the morning, the west is on his right, so he is facing south.

Example 1: In the morning, Asha's shadow falls exactly to her left. Which way is she facing?

1. Morning shadows point west.
   Why: The sun rises in the east.
2. West is on her left.
3. She faces north.

Answer: North

Formula summary

`Right turn`	N → E → S → W → N (clockwise).
`Left turn`	N → W → S → E → N.
`Shortest distance`	√(net east^2 + net north^2).
`Shadow model`	Morning sun in the east → shadow to the WEST. Evening → shadow to the EAST.
`Angles`	Clockwise 90 = right, 180 = about-turn. 45 lands on NE/SE/SW/NW.

Shortcuts

Net displacement Use when: Several legs, then 'how far from start'.

1. Add all N-S legs, add all E-W legs separately.
2. Then one Pythagoras.

Try it: Walk 3 km N, 4 km E. Distance from start?

1. 3-4-5.

Answer: 5 km

Check yourself

- You can draw every walk.
- You can turn right = clockwise.
- You can use Pythagoras for the straight-line distance.
- You can use sunrise-east for shadows.

Class plan (2 hours)

`10 min`	Warm-up.

30 min	Compass and turns: draw 5 walks.
20 min	Distance with Pythagoras; final direction.
15 min	Shadows.
40 min	Practice set.
5 min	Recap.

Practice: 6 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/directions

Analogy

> **Doctor is to hospital as teacher is to ? School. An analogy asks you to find the relation in the first pair and repeat it.**

Word, number and letter analogies test one skill: say the relationship in a sentence, then apply the same sentence.

Say the relationship as a sentence

'A doctor works in a hospital.' Now: 'A teacher works in a school.' If the sentence fits more than one option, make it more specific.

Common relations: worker-place, tool-worker, part-whole, cause-effect, opposite, young one (dog : puppy), group (fish : shoal).

Example 1: Pen : Writer :: Scalpel : ?

1. 'A writer uses a pen.'
   Why: Tool - user.
2. A surgeon uses a scalpel.

Answer: Surgeon

Number and letter analogies

Numbers: look for square, cube, x 2 + 1, sum of digits. 4 : 64 :: 5 : 125 (cube).

Letters: compare positions. ACE : BDF means each letter +1.

Example 1: 7 : 50 :: 9 : ?

1. $7^2 + 1 = 50$.
   Why: Try squares first.
2. $9^2 + 1 = 82$.

Answer: 82

Example 2: CAT : DBU :: DOG : ?

1. Each letter +1.
2. EPH.

Answer: EPH

Formula summary

`Word analogy`	Worker:tool, product:raw material, part:whole, animal:young, synonym, antonym.
`Number analogy`	Square, cube, x2+1, digit sum. 3:27 :: 4:64 (cube).

`Letter analogy`	Same shift on each letter. ACE:BDF :: GIK:HJL.

Shortcuts

Number-pair check Use when: 3 : 10 :: 5 : ?

1. Test $n^2 + 1$: 9 + 1 = 10.
2. 5 → 26.

Try it: 3 : 10 :: 5 : ?

1. $n^2 + 1$.

Answer: 26

Check yourself

- You can say the relation as a sentence.
- You can test squares and cubes for number pairs.
- You can compare letter positions.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Word relations: 10 pairs, say the sentence.
`25 min`	Number analogies.
`15 min`	Letter analogies.
`35 min`	Practice set.
`5 min`	Recap.

Practice: 6 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/analogy

Ranking

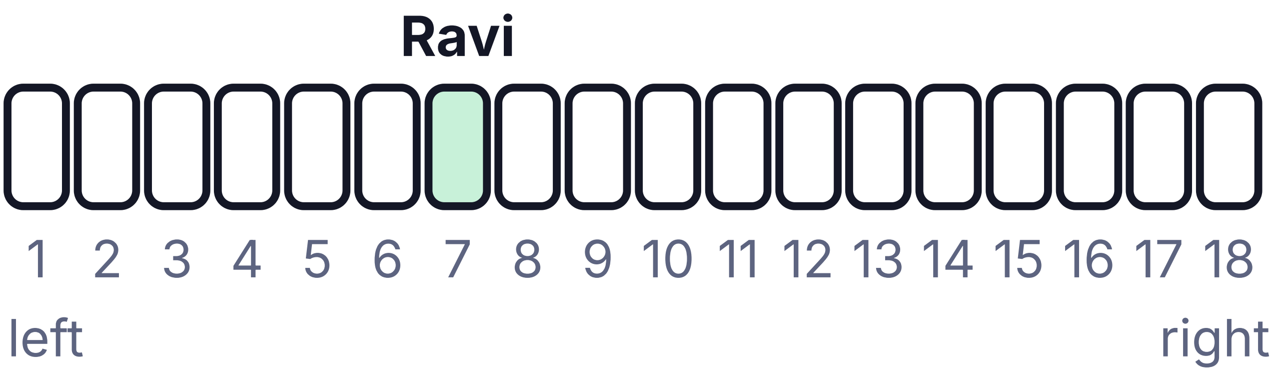
> **Ravi is 7th from the left and 12th from the right in a row. How many people are there? 7 + 12 - 1 = 18: Ravi was counted twice.**

Ranking questions all come from one identity: position from left + position from right = total + 1.

Left + right - 1 = total

A person counted from both ends is counted twice, so subtract 1.

Between two people: if A is p-th from the left and B is q-th from the left, there are $|p - q| - 1$ people between them.



7th from the left and 12th from the right: 18 people

`Total = left rank + right rank - 1`	
`Right rank = total - left rank + 1`	

Example 1: In a class of 40, Priya is 15th from the top. What is her rank from the bottom?

1. Bottom rank = 40 - 15 + 1.
   Why: Rearranged identity.
2. = 26.

Answer: 26

Example 2: In a row, A is 10th from the left and B is 15th from the right. If they swap places, A becomes 18th from the left. How many people are in the row?

1. After the swap A is where B was: 18th from the left and 15th from the right.
   Why: The swap tells us B's left position.
2. 18 + 15 - 1 = 32.

Answer: 32

Formula summary

`Total`	Left rank + right rank - 1.
`Swap two people`	The change in one's rank equals the change in the other's.
`Between two people`	Total - (sum of their ranks from opposite ends) ... draw it when overlap is possible.

Shortcuts

Total from both ends Use when: '15th from the top and 20th from the bottom'.

1. 15 + 20 - 1 = 34.

Try it: Total students?

1. 34.

Answer: 34

Check yourself

- You can use left + right - 1 = total.
- You can count people between two positions.
- You can solve swap questions.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	The identity with a real row of students.
`25 min`	People between two positions.
`15 min`	Swaps.
`35 min`	Practice set.
`5 min`	Recap.

Practice: 6 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/ranking

Syllogisms

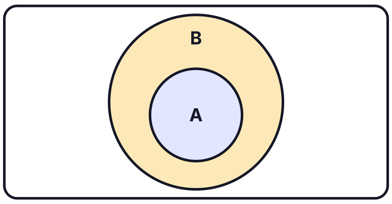
> **All cats are animals. Some animals are black. Does it follow that some cats are black? No. Draw the circles and you can see why.**

Syllogisms are decided by pictures, not by common sense. The statements may be silly ('All pens are dogs'); treat them as true and draw.

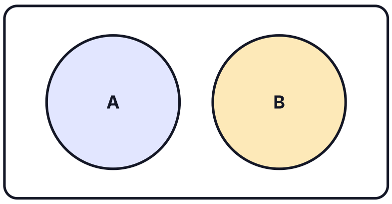
Four kinds of statement, four pictures

All A are B: the A circle is inside B. No A is B: the circles do not touch. Some A are B: the circles overlap. Some A are not B: part of A is outside B.

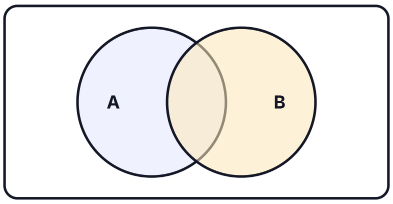
A conclusion follows only if it is true in EVERY way you could draw the circles. If you can draw one picture where it is false, it does not follow.



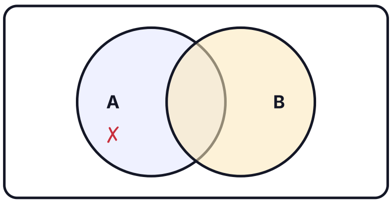
All A are B: the A circle sits inside the B circle



No A is B: the circles do not touch



Some A are B: the circles overlap (at least one A is a B)



Some A are not B: part of A lies outside B

Example 1: Statements: All cats are animals. Some animals are black. Conclusion: Some cats are black. Does it follow?

1. Draw cats inside animals.
   Why: First statement.
2. Draw 'black' overlapping animals, but outside the cat circle.
   Why: Allowed: 'some animals are black' does not say which animals.
3. In that picture no cat is black, so the conclusion does not follow.

Answer: **Does not follow**

'Either-or' and possibility

If two conclusions are 'Some A are B' and 'No A is B', one of them must be true even when neither follows alone. The answer is 'either I or II follows'.

'Some A being B is a possibility' follows whenever the circles are allowed to overlap (nothing says 'No A is B').

Example 1: Statements: Some pens are books. No book is a bag. Conclusion: Some pens are not bags. Does it follow?

1. The pens that are books cannot be bags (no book is a bag).
   Why: Follow the overlap.
2. So at least those pens are not bags. It follows.

Answer: Follows

Formula summary

`All A are B`	A circle fully inside B.
`Some A are B`	Circles overlap (at least a bit).
`No A is B`	Circles apart.
`All + All = All`	All A are B, all B are C → all A are C.
`All + No = No`	All A are B, no B is C → no A is C.
`Some + All = Some`	Some A are B, all B are C → some A are C.
`Possibility`	'Some A being C is a possibility' follows whenever it is not ruled out by a 'No'.
`Either-or`	When two conclusions are 'Some X are Y' and 'No X is Y' and neither follows alone, 'either' follows.

Shortcuts

Break the conclusion Use when: To test a conclusion, try to draw a diagram where it is FALSE.

1. If you can, it does not follow.
2. If you cannot, it follows.

Try it: All cats are dogs. Some dogs are birds. Conclusion: some cats are birds?

1. Draw birds overlapping dogs but not the cat circle.
2. Possible, so it does not follow.

Answer: Does not follow

Check yourself

- You can draw the four statement types.

- You can accept a conclusion only if no picture breaks it.
- You can spot either-or pairs.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Four pictures; draw each statement.
`25 min`	Testing conclusions: find one picture that breaks it.
`15 min`	Either-or and possibility.
`35 min`	Practice set.
`5 min`	Recap.

Practice: 5 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/syllogisms

Seating Arrangement

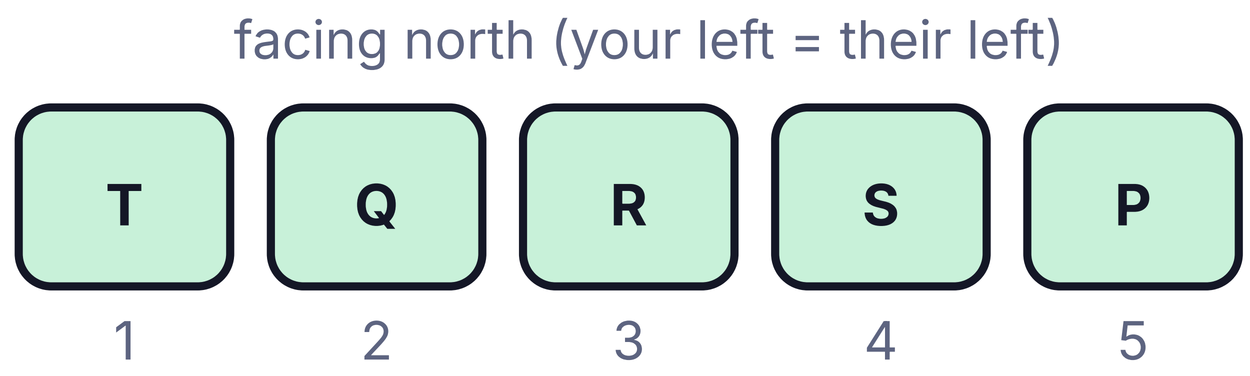
> **Seating puzzles look long, but they are just clues placed one at a time. Start with the clue that fixes a seat, and draw after every clue.**

Seating sets carry 3 to 5 questions. A clean drawing method turns a 5-minute puzzle into 2 minutes.

Linear: facing north or south

Number the seats from the left of your page. If people face north, their left is your left. If they face south, their left is YOUR right.

'Immediately right' means the next seat. 'Second to the right' means skip one.



T Q R S P facing north: R is in the middle

Example 1: P, Q, R, S, T sit in a row facing north. R is in the middle. P is at the right end. S is immediately left of R. Q is not at an end. Who is at the left end?

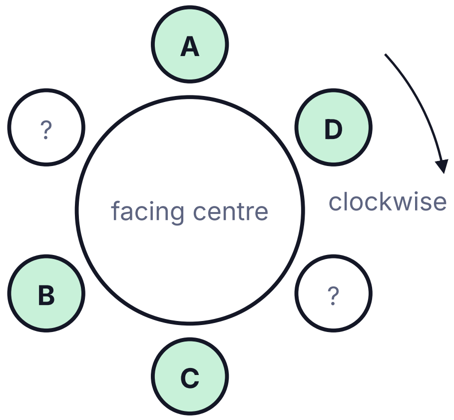
1. Seats 1 to 5 from the left. R = 3, P = 5.
   Why: Fixed seats first.
2. S is immediately left of R: S = 2.
   Why: Facing north, left is your left.
3. Q is not at an end: Q = 4.
   Why: Seats 1 and 5 are the ends.
4. T takes seat 1.

Answer: T

Circular: facing the centre

Put the most mentioned person at the top. For people facing the centre, their right is anticlockwise on your drawing, their left is clockwise.

In a table of n (n even), the person opposite is n/2 seats away.



The example below: A at the top, D on A's left (clockwise), B second to A's right (anticlockwise), C opposite A

Example 1: Six people sit around a table facing the centre. A is at the top. B is second to the right of A, C is opposite A, D is immediately left of A. Who is opposite B?

1. A at the top. Right of A (facing centre) is anticlockwise.
   Why: Draw the circle first.
2. C opposite A. D one seat clockwise from A. B two seats anticlockwise.
3. The seat opposite B is the one just clockwise of A, which is D.

Answer: **D**

Formula summary

`Linear, facing north`	Your left = the reader's left.
`Circular, facing centre`	Seen from above: your left is clockwise, your right is anticlockwise.
`Immediate neighbours`	'Sits second to the right' means skip one seat.
`Count check`	n people round a table: each has exactly 2 neighbours; opposite exists only when n is even.

Shortcuts

Anchor and branch Use when: When a clue allows two places, draw BOTH sketches and let a later clue kill one.

1. Place the fixed clue first (a named seat, an end).
2. When a clue gives two options, copy the sketch and try each.
3. Apply the remaining clues to both sketches. Cross out the one that breaks a clue.

Try it: P, Q, R, S, T sit in a row facing north. Q is second from the left. R sits next to Q. T is at one end. S is not at an end, and P sits immediately to the right of S. R is not at an end. Who is in the middle?

1. Number the seats 1 to 5 from the left. Q = seat 2.
   Why: 'Second from the left' is a fixed seat, so it is the anchor.
2. R is next to Q, so R = 1 or R = 3. Draw two sketches: (a) R Q _ _ _ and (b) _ Q R _ _.
   Why: The clue allows two places, so we keep both alive instead of guessing.
3. 'R is not at an end' kills sketch (a), because seat 1 is an end.
   Why: A later clue removes one branch; that is the whole trick.
4. In sketch (b), S is not at an end and P is right after S, so S = 4 and P = 5.
   Why: Seats 1, 2, 3 are either taken or an end; S = 4 is the only seat left that is not an end and has a free seat to its right.
5. T takes the last free seat, 1, which is an end, as the clue says. Row: T Q R S P.
   Why: Always finish by checking every clue against the final row.
6. The middle seat (3) is R.

Answer: R

Circle: fix one person, then count Use when: Round-table questions. Fix any one person at the top; the circle has no 'first seat' otherwise.

1. Draw the circle with n seats and put the most mentioned person at the top.
2. Facing the centre: their right is anticlockwise on your drawing, their left is clockwise.
3. 'Second to the right' means move two seats, not one.

Try it: A, B, C, D, E, F sit around a round table facing the centre, equally spaced. B is second to the right of A. C is opposite A. D is immediately left of A. E is not next to C. Who sits opposite D?

1. Six seats, numbered 0 to 5 anticlockwise on paper. Put A at seat 0.
   Why: Facing the centre, a person's right is anticlockwise when you look down on the table.
2. B is second to the right of A: seat 2. C is opposite A: seat 3.
   Why: Opposite in a table of 6 is 3 seats away.
3. D is immediately left of A: one step clockwise, which is seat 5.
   Why: Left of a centre-facing person is clockwise.
4. E and F take seats 1 and 4. Seat 4 is next to C (seat 3), so E = 1 and F = 4.
   Why: 'E is not next to C' decides which of the two free seats E gets.
5. Opposite D (seat 5) is seat 2, which is B.

Answer: B

Common mistakes

Wrong: Treating 'second to the right' as the next seat.	**Instead:** Skip one seat.

Wrong: Forgetting to mirror left and right for people facing south.	**Instead:** Their left is your right.

Check yourself

- You can draw after every clue.
- You can turn left/right correctly for north, south and centre facing.
- You can keep two sketches when a clue allows two places.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Linear: north and south facing, with the row picture.
`30 min`	Circular: centre-facing, opposite, second to the right.
`15 min`	Anchor and branch: keep two sketches alive.
`30 min`	Practice set.
`5 min`	Recap.

Practice: 3 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/seating-arrangement

Statements: Assumptions and Conclusions

> **'Buy our soap for glowing skin.' The advertiser assumes people want glowing skin. An assumption is what the speaker must believe for the statement to make sense.**

These questions test careful reading, not maths. The rules are strict: an assumption is unstated and necessary; a conclusion follows from the statement alone.

Assumptions

An assumption is something the speaker takes for granted without saying it. Test: if the assumption were false, would the statement still make sense? If not, it is assumed.

Reject anything that is merely possible, or too extreme ('only', 'always', 'all').

Example 1: Statement: 'Please send your CV by email.' Assumption I: the reader has access to email. Assumption II: CVs sent by post will be rejected.

1. If the reader had no email, the request would make no sense, so I is assumed.
   Why: Negation test.
2. Nothing says post is rejected; II goes beyond the statement.
3. Only I is implicit.

Answer: Only I

Conclusions

A conclusion must follow from the statement alone, without outside knowledge. Stay inside the facts given.

Watch definite words: 'all', 'never', 'must'. A conclusion using them needs statement support for them.

Example 1: Statement: 'Most students of this college got placed this year.' Conclusion I: some students were not placed. Conclusion II: the college has good teachers.

1. 'Most' means not all, so some were not placed.
   Why: I follows from the words.
2. Nothing is said about teachers.
   Why: II needs outside knowledge.
3. Only I follows.

Answer: Only I

Formula summary

`Assumption test`	If the assumption were false, would the statement still make sense? If not, it is implicit.
`Conclusion test`	Must be true for ANY reader who accepts the statement. 'May', 'always', 'only' usually break it.

`Advertisement statements`	Assume people read ads and the product is worth buying.
`Extreme words`	'Only', 'all', 'never', 'best' in an option usually make it wrong.

Shortcuts

Negation test Use when: Deciding if an assumption is implicit.

1. Negate it and re-read the statement.

Try it: 'Use Brand X soap for soft skin.' Assumption: people want soft skin.

1. If nobody wanted soft skin the ad is pointless.
2. So it is implicit.

Answer: **Implicit**

Check yourself

- You can test an assumption by negating it.
- You can accept only conclusions that use the statement's facts.
- You can reject extreme words without support.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Assumptions with the negation test.
`30 min`	Conclusions: inside the facts.
`45 min`	Practice set with discussion.
`5 min`	Recap.

Practice: 3 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/statements-assumptions-and-conclusions

Puzzles

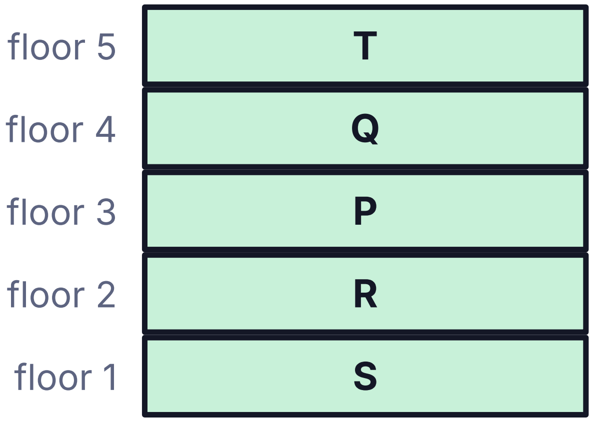
> **Floor puzzles and number puzzles are seating puzzles in another shape. Build a grid and fill it clue by clue.**

Puzzle sets carry many marks. A table with one row per person or floor keeps every clue visible.

Floor puzzles: build the building

Draw the floors with 1 at the bottom (unless the question says the top floor is 1). Place the fixed clues first ('P lives on floor 3'), then the relative ones ('Q lives just above R').

Keep two versions when a clue allows two positions, and let the next clue decide.



Five floors, one person each: S R P Q T from the bottom

Example 1: Five people P, Q, R, S, T live on floors 1 to 5 (1 at the bottom), one per floor. P is on floor 3. Q lives immediately above P. R lives immediately below P. S is not on the top floor. Who lives on floor 5?

1. P = 3, Q = 4, R = 2.
   Why: Fixed clue first, then the two neighbours.
2. Floors 1 and 5 remain for S and T. S is not on top, so S = 1.
3. T lives on floor 5.

Answer: T

Formula summary

`Grid`	Rows = people, columns = floors/days/colours.
`Definite first`	Place 'X lives on the top floor' clues before 'X is above Y' clues.
`Floor puzzles`	Floor 1 = bottom unless told otherwise. 'Above' does not mean 'immediately above'.

`Number puzzles`	Set up equations: sum, difference, product - then check options.

Shortcuts

Floor puzzle order Use when: 5 people, 5 floors.

1. Write floors 5 to 1 vertically.
2. Place the fixed ones, then the 'immediately above/below', then the rest.

Try it: A is on the top floor. C is immediately below A. E is on floor 1. B is above D. Who is on floor 3?

1. A5, C4, E1.
2. B above D in floors 2-3 → B3, D2.

Answer: B

Check yourself

- You can build a table before reading the second clue.
- You can place fixed clues first.
- You can branch when a clue allows two positions.

Class plan (2 hours)

`10 min`	Warm-up.
`35 min`	Floor puzzles with a table.
`30 min`	Two-attribute puzzles (person, city, job) with a grid.
`40 min`	Practice set.
`5 min`	Recap.

Practice: 4 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/puzzles

Odd One Out / Classification

> **Apple, mango, carrot, banana: carrot is the odd one because it is a vegetable. Find the rule three of them share.**

Classification is quick marks: find a property that three options share and one does not.

Numbers and letters

Numbers: test prime or not, square or cube, even or odd, digit sum, divisible by something.

Letters: convert to positions and test gaps, vowels, opposite pairs.

Example 1: Find the odd one: 27, 64, 125, 144, 216

1. $27 = 3^3$, $64 = 4^3$, $125 = 5^3$, $216 = 6^3$.
   Why: Cubes.
2. $144 = 12^2$ is not a cube.

Answer: 144

Example 2: Find the odd one: BD, FH, JL, NQ

1. BD: gap 2. FH: 2. JL: 2. NQ: 3.
   Why: Letter positions.
2. NQ.

Answer: NQ

Formula summary

`Numbers`	Prime / composite, square / cube, divisible by, digit sum.
`Letters`	Gaps between letters, vowels, reverse pairs.
`Words`	Category (fruit, tool), function, part-whole.

Shortcuts

Letter-gap check Use when: Letter groups.

1. Convert to positions and look at the gaps.

Try it: ACE, BDF, GIK, LNQ

1. Gaps 2,2 in the first three; LNQ is 2,3.

Answer: LNQ

Check yourself

- You can test prime, square, cube, parity and digit sum.
- You can turn letters into positions and compare gaps.

Class plan (2 hours)

`10 min`	Warm-up.
`35 min`	Number properties checklist.
`30 min`	Letter groups by position.
`40 min`	Practice set.
`5 min`	Recap.

Practice: 5 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/odd-one-out-classification

Blood Relations

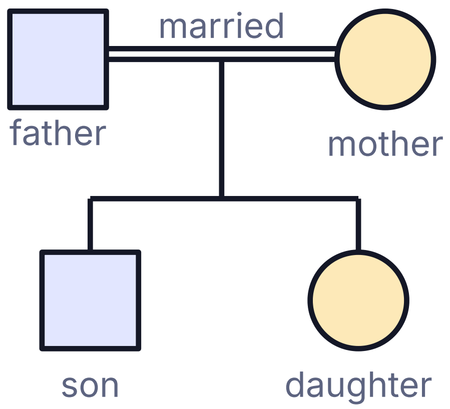
> **'Pointing to a man, Rani says, his mother is my mother's only daughter.' Rani's mother's only daughter is Rani herself. So the man is Rani's son. Break the sentence from the end.**

Blood relation questions become easy with a family tree and a fixed set of symbols. Always decode from the end of the sentence.

Draw a family tree

Square for male, circle for female, a double line for marriage, a vertical line down to children. Put each generation on its own level.

Mother's or father's brother = uncle, sister = aunt. Son's wife = daughter-in-law. Brother's son = nephew.



Family tree symbols: square = male, circle = female, = marriage, vertical line = child

Example 1: Pointing to a man, Rani says: 'His mother is my mother's only daughter.' How is the man related to Rani?

1. 'My mother's only daughter' is Rani herself.
   Why: Start from the end of the sentence.
2. So the man's mother is Rani.
3. The man is Rani's son.

Answer: **Son**

Example 2: A is B's brother. C is A's mother. D is C's father. How is B related to D?

1. A and B are siblings, both children of C.
   Why: Brothers share parents.
2. D is C's father, so D is the grandfather of A and B.
   Why: Two generations up.
3. B is D's grandchild.

Answer: **Grandchild**

Coded relations

'A + B means A is the son of B, A - B means A is the wife of B ...'. Decode each symbol into a small tree, then join the trees.

Example 1: P # Q means P is the brother of Q; P @ Q means P is the mother of Q. What does X @ Y # Z mean?

1. Y # Z: Y is Z's brother, so Y and Z share parents.
   Why: Decode each symbol.
2. X @ Y: X is Y's mother.
3. So X is Z's mother too.

Answer: X is Z's mother

Formula summary

`Father's / mother's brother`	Uncle (paternal / maternal).
`Brother's / sister's son`	Nephew; daughter → niece.
`Father's father`	Grandfather. Son's wife → daughter-in-law.
`'Only son of my father'`	The speaker himself.
`Coded relations`	A + B = A is father of B etc. Translate each symbol then draw.

Shortcuts

Pointing / introduction questions Use when: 'Pointing at a man, Rita said, "His mother is the only daughter of my mother."'

1. Only daughter of my mother = Rita herself.
2. So Rita is his mother.

Try it: How is the man related to Rita?

1. Son.

Answer: Son

Check yourself

- You can draw a tree with fixed symbols.
- You can decode 'pointing' sentences from the end.
- You can turn coded symbols into small trees and join them.

Class plan (2 hours)

`10 min`	Warm-up.
`25 min`	Symbols and trees.

25 min	Pointing-to-a-photo questions from the end.
20 min	Coded relations.
35 min	Practice set.
5 min	Recap.

Practice: 4 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/blood-relations

Counting Figures

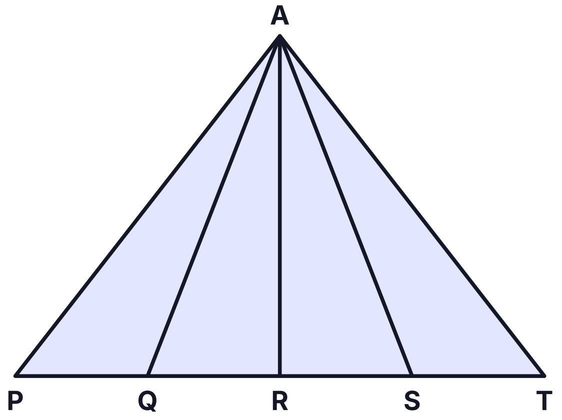
> **How many triangles are in a triangle with three lines drawn from the top? Not 4. Count by size, and it is 10.**

Counting figures looks visual but it is really counting in a system. Label every point and count shapes by how many small pieces they use.

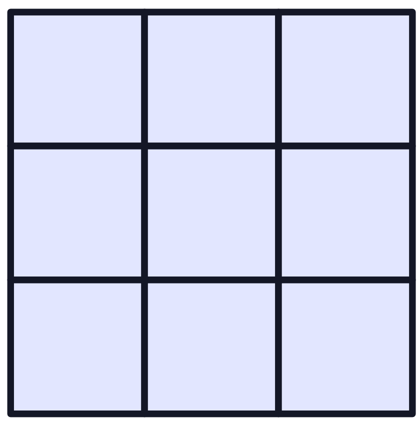
Label, then count by size

Label every corner and meeting point with a letter. Count the smallest shapes first, then shapes made of 2 pieces, 3 pieces and so on.

Triangle split by n lines from the top corner: the base has n + 1 pieces, and the number of triangles = (n + 1)(n + 2)/2.



Count by size: 4 single pieces, 3 made of two, 2 of three, 1 of four = 10



A 3 × 3 grid has 9 + 4 + 1 = 14 squares

`Lines from the apex splitting the base into k parts`	Triangles = k(k + 1)/2
`Squares in an n × n grid`	$1^2 + 2^2 + ... + n^2$
`Rectangles in an m × n grid`	(m+1)C2 × (n+1)C2

Example 1: A triangle has 3 lines drawn from the top corner to the base (4 small triangles). How many triangles in total?

1. Base split into 4 parts.
   Why: Each triangle is picked by choosing two points on the base.
2. Choose 2 of the 5 base points: 5C2 = 10.
   Why: Every pair of base points with the apex makes a triangle.
3. 10.

Answer: 10

Example 2: How many squares are there on a chessboard (8×8)?

1. 1×1: 64, 2×2: 49, 3×3: 36 ... 8×8: 1.
   Why: An $n \times n$ grid has $(n - k + 1)^2$ squares of size k.
2. $1 + 4 + 9 + ... + 64 = 204$.

Answer: 204

Formula summary

Squares in an n × n grid	$1^2 + 2^2 + ... + n^2 = n(n + 1)(2n + 1)/6$.
Rectangles in an m × n grid	$C(m + 1, 2) \times C(n + 1, 2)$.
Triangle with n lines from one vertex to the base	$n(n + 1)/2$ triangles (n lines including sides).
Triangles in a triangle divided into n rows (all upright + inverted)	For n = 1, 2, 3, 4: 1, 5, 13, 27.

Shortcuts

Squares in a chessboard Use when: 8×8.

1. $1 + 4 + 9 + ... + 64 = 204$.

Try it: Squares on a chessboard?

1. $8 \times 9 \times 17 / 6$.

Answer: 204

Check yourself

- You can label every point before counting.
- You can count by number of pieces.
- You can use the grid formulas.

Class plan (2 hours)

10 min	Warm-up.
30 min	Labelling and counting by size.
25 min	Formulas for triangles, squares, rectangles.
15 min	Mixed figures.
35 min	Practice set.
5 min	Recap.

Practice: 7 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/counting-figures

LOGICAL REASONING · DAY 12

Cryptarithmetic

> **SEND + MORE = MONEY: every letter is a different digit. You solve it like a detective: the carry into a new column can only be 1.**

Cryptarithm questions reward a few logical starting moves. After that, trial with the remaining digits is quick.

Starting moves

The sum of two numbers can carry at most 1 into a new column. If the answer is one digit longer than both numbers, its first letter is 1.

A letter that starts a number is never 0. In a column where X + X ends in X, X is 0 (or 9 with a carry).

Work from the leftmost column and use carries to limit the choices, then fill the rest by trial.

```
    S E N D
+   M O R E
-----------
  M O N E Y
```

SEND + MORE = MONEY: M must be 1

Example 1: In SEND + MORE = MONEY, what digit is M?

1. Two 4-digit numbers add to a 5-digit number.
   Why: The extra digit comes from a carry.
2. A carry out of the top column can only be 1.
   Why: Even 9999 + 9999 = 19998.
3. M = 1.

Answer: 1

Example 2: AB + BA = 121, where A and B are digits. Find A + B.

1. AB + BA = (10A + B) + (10B + A) = 11(A + B).
   Why: Write the two-digit numbers in place value.
2. 11(A + B) = 121, so A + B = 11.

Answer: 11

Formula summary

`Leading carry`	When two n-digit numbers give an (n + 1)-digit sum, the new front digit is 1.
`Same letter, same digit`	Different letters, different digits. Leading letters are not 0.
`Column rule`	Digit + digit + carry-in = result + 10 × carry-out.
`X + X = X`	Then X = 0 (or X = 9 with carry-in 1 and a carry out).

Shortcuts

Classic: SEND + MORE = MONEY Use when: The standard 8-letter puzzle.

1. M = 1 (leading carry).
2. S + 1 (+carry) ≥ 10 and gives O → S = 9, O = 0.
3. Then E, N, D, R, Y follow column by column.

Try it: SEND + MORE = MONEY. Value of MONEY?

1. 9567 + 1085 = 10652.

Answer: 10652

Check yourself

- You can set the leading carry to 1.
- You can never put 0 at the front.
- You can use place value for two-digit letter numbers.

Class plan (2 hours)

`10 min`	Warm-up.
`30 min`	Starting moves: leading 1, no leading zero, carries.
`30 min`	SEND + MORE worked on the board.
`15 min`	Place-value tricks (AB + BA).
`30 min`	Practice set.
`5 min`	Recap.

Practice: 4 questions in the Practice Book and at aptitude.getmaxsolutions.com/practice/cryptarithmetic